Volatility During the Financial Crisis Through the Lens of High Frequency Data: A Realized GARCH Approach

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Abstract

We study the financial volatility during the global financial crisis and use the largest volatility shocks to identify major events during the crisis. Our analysis makes extensive use of high-frequency (HF) financial data for the modeling of volatility and, importantly, for determining the timing within the day when the largest volatility shocks occurred. The latter helps us identify the event(s) that can be associated with each of these shocks, and serves to illustrate the benefits of using high-frequency data. Some of the largest volatility shocks coincide, not surprisingly, with the bankruptcy of Lehman Brothers on September 15, 2008 and Congress's failure to pass the Emergency Economic Stabilization Act on September 29, 2008. The day with the largest volatility shock was February 27, 2007 – a date where Freddie Mac announced a stricter policy for underwriting subprime loans and a date that was marked by a crash on the Chinese stock market. However, the intraday HF data shows that the main culprit for this shock was a computer glitch in the trading system. On the other hand, the days with the largest drops in volatility can in most cases be related to interventions by governments and central banks.

Keywords: Financial Crisis; Volatility; High Frequency Data; Realized GARCH.
JEL Classification: C10; C22; C80

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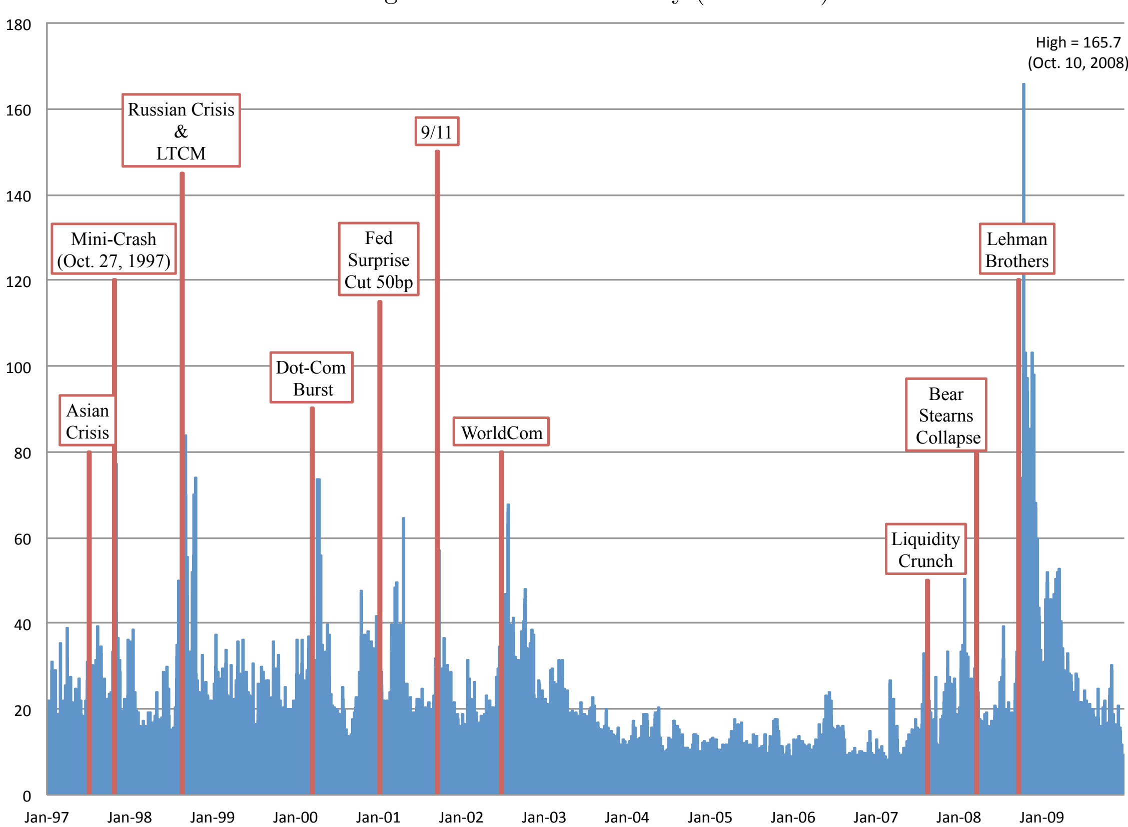
1 Introduction

Financial crises and stock market volatility are closely linked. Their driving forces have varied over time from changes in fundamentals to technical and psychological factors. In the recent crisis, the latter category has dethroned fundamental data through the rise of quantitative trading and the availability of high-frequency financial data and psychological perception. Nowadays, uncertainty is driven by real events, since there are lots of differences of opinion among market participants who are always trying to figure out how the situation will impact the future. Consequently, it is essentially the news that causes volatility.

The aim of this paper is primarily to study the financial volatility during the global financial crisis. We use the largest shocks on volatility to identify the major events during the crisis, as well as their causes. The available sample of financial data spans the period from 1997 to 2009 that includes several international crises and major events such as 9/11. This adds perspective to the magnitude of the global financial crisis. The tight relationship between important financial/economic events and volatility is illustrated in Figure 1 that presents the annualized realized measure of volatility for the S&P 500 index covering the period 1997-2009. Several important clusters of volatility are observed and associated with economic and/or financial events that occurred at that time (*e.g.*, Asian crisis, Russian crisis, Dot-com bubble burst, 9/11, Lehman Brothers collapse, etc.). The highest value of the volatility measure is recorded during the recent global crisis (on 10 October, 2008), in a period of high financial stress.

Figure 1: Realized volatility (1997-2009)



Note: This figure displays the annualized realized volatility for the period 1997-2009 and the time of some of the major crises and events.

We utilize the recently developed Realized GARCH framework (Hansen et al., 2012) to extract daily volatilities. This framework also provides us with measures of volatility shocks. Most importantly, we propose a robustified Realized GARCH model that is appropriate in this context with large shocks. Our model is less sensitive to outliers, improving hence the empirical fit during the crisis period. The Robust Realized GARCH model makes use of high-frequency data and deals with both the asymmetry and the impact of outliers on volatility. The new model behaves better than classical approaches in terms of the treatment of extreme shocks on volatility and the adjustment of their persistence in time.

The knowledge of financial volatility has considerably increased over the last decade, revolving around two main lines: measuring and forecasting volatility dynamics. This is in part due to the upraised research interest in this topic and the availability of high-quality financial market data. The access to high-frequency financial data has indeed inspired the development of new econometric and statistical tools that substantially improved the ex-post volatility measurement and forecast evaluation.

First, high-frequency data are used to provide accurate proxies of the underlying daily volatility. The vastly growing literature on measuring volatility was largely spurred by Andersen and Bollerslev (1998), who documented that the realized variance computed as the sum of squared intraday returns provides an accurate measurement of daily volatility. The stochastic properties of the realized variance were subsequently studied in Andersen et al. (2001), Barndorff-Nielsen and Shephard (2002), Meddahi (2002), Andersen et al. (2003), Mykland and Zhang (2009). In the meanwhile, a large number of improved proxies of volatility that are not sensitive to market microstructure noise (*e.g.*, realized bipower variation, realized kernel, etc.) have been introduced by Barndorff-Nielsen and Shephard (2004), Zhang (2006), Barndorff-Nielsen et al. (2008), Hansen and Horel (2009), *inter alios*. Second, the realized measures are far more enlightening about the current level of volatility than the squared returns, which motivates their use in volatility modeling.

Creal et al. (2011, 2013) and Harvey (2013) have also proposed a new specific class of models (generally called dynamic conditional score (DCS) or generalized autoregressive score (GAS) models), that encompasses many of the existing observation-driven models. These models are not dealing with realized measures of volatility, but they allow parameters to change over time using the score function of the conditional distribution of the observations. An immediate practical advantage of taking into account the score function comes from the response of the score to the outliers.

Inspired by the existing literature on volatility, we econometrically improve the Realized EGARCH model discussed in Hansen and Huang (2012) by integrating the score of a higher tailed distribution.[1] The score function is thus used to transform the innovation terms both in the conditional volatility and realized measure equations and hence dampen the influence of the outliers on volatility. The new model is well suited for situations where volatility exhibits jumps to a new level over a short period of time, as the conditional volatility will be able to rapidly reach this new level.

For a more focused analysis, we zoom in on the events during the recent global crisis (2007-2009) and analyze the days with the largest volatility shocks. We present then the main economic/financial/social/

[1] The main difference between the Realized GARCH and Realized EGARCH models is given by the presence (in the latter) of two innovations in the conditional volatility equation accounting for the shocks coming both from the returns and the realized measure of volatility.

governmental events that could have induced these shocks. We subsequently use the information in the high-frequency data to identify the exact timing of the shock that gives us an idea about its real cause. Interestingly, the largest volatility shock is found to coincide with a technical problem in the trading system.

The paper is organized as follows. Section 2 introduces the modeling framework, focusing on the Robust Realized GARCH specification. The empirical analysis is presented in Section 3. In Section 4 we discuss the news related to the largest volatility shocks. Section 5 concludes.

2 Modeling Framework

2.1 Key Variables

We are to study volatility of asset returns, r_t. In our empirical analysis we use the exchange traded index fund, SPY, to define daily returns because it closely tracks the S&P 500 index and provides us with readily available high-frequency data. The conditional variance of daily returns is given by

$$h_t = \mathrm{var}(r_t|\mathcal{F}_{t-1}), \tag{1}$$

where $\{\mathcal{F}_t\}$ is a filtration to which r_t is adapted. Volatility shocks – the key variable in our analysis – are defined by

$$v_t = \mathrm{E}(\log h_{t+1}|\mathcal{F}_t) - \mathrm{E}(\log h_{t+1}|\mathcal{F}_{t-1}), \tag{2}$$

so that $100 \times v_t$ is the percentage shock to volatility, induced by news on the t^{th} day.

In the rest of this section we detail the econometric modeling of returns and realized measures of volatility, that produce the empirical estimates of volatility shocks. Readers who are primarily interested in the empirical analysis and less interested in the details of the econometric models can skip the rest of this section and go immediately to Section 3.

2.2 Realized GARCH Framework

The Realized EGARCH model of Hansen and Huang (2012) (with a single realized measure of volatility) is given by the following three equations:

$$\begin{aligned} r_t &= \mu + \sqrt{h_t} z_t, && (3)\\ \log h_t &= \omega + \beta \log h_{t-1} + \tau(z_{t-1}) + \gamma u_{t-1} && (4)\\ \log x_t &= \xi + \varphi \log h_t + \delta(z_t) + u_t, && (5) \end{aligned}$$

where $\tau(z) = \tau_1 z + \tau_2(z^2 - 1)$ and $\delta(z) = \delta_1 z + \delta_2(z^2 - 1)$. Here, z_t and u_t are typically assumed to be mutually and serially independent and modeled with a Gaussian specification: $z_t \sim \mathrm{iid}N(0, 1)$ and $u_t \sim \mathrm{iid}N(0, \sigma_u^2)$.[2]

[2]The Gaussian specifications can be motivated by the empirical findings in Andersen, Bollerslev, Diebold and Labys (2001), Andersen, Bollerslev, Diebold and Ebens (2001) and Andersen et al. (2003) who show that realized volatility is

The three equations are labelled as the *return equation,* the *GARCH equation*, and the *measurement equation,* respectively. The first two (alone) form the basis for a GARCH-X model, similar to that estimated by Engle (2002), Barndorff-Nielsen and Shephard (2007), and Visser (2011). The measurement equation is a key characteristic of the Realized GARCH framework, that ties the (ex-post) realized measure, x_t, to the latent (ex-ante) conditional variance, h_t. A GARCH-X model is – in isolation – an incomplete description of the data, because it does not model the realized measure. A complete specification of the dynamic properties of both returns and realized measures is achieved by means of the measurement equation. Other approaches to completing the GARCH-X model were proposed by Engle and Gallo (2006) and Shephard and Sheppard (2010).

Some of the key features of this model are captured by β, which measures the persistence of volatility, and by $\tau(z_{t-1}) + \gamma u_{t-1}$ that estimates the innovation in the conditional volatility. For instance, γu_{t-1} captures the impact that (a shock to) the realized measure has on the next period conditional variance. The functions $\tau(z)$ and $\delta(z)$ are called the leverage functions, as they specify a dependence between returns and volatility that is commonly referred to as the *leverage effect.* Hansen et al. (2012) explored different leverage functions and found a simple quadratic form to be satisfactory in practice. We adopt the same structure in our estimation. In addition, the term $\tau(z)$ makes reference to the *news impact curve* introduced by Engle and Ng (1993), that maps out how positive and negative returns impact future volatility.

2.3 Robustified Realized GARCH

The global financial crisis period includes several large shocks to returns and volatility. Conventional GARCH models are sometimes found to be too sensitive to large returns which motivated Harvey (2013) to suggest a more robust dynamic structure that utilizes the conditional scores of the model. This type of models are known as dynamic conditional score (DCS) or generalized autoregressive score (GAS) models, see Creal et al. (2011, 2013).

In the present context with realized GARCH models, we adopt some ideas from Harvey (2013) by introducing parameters that serve to dampen the impact of outliers. For instance, we substitute $\tilde{z}_t = z_t/(1 + z_t^2/d)$ for z_t in the GARCH equation where d is a parameter to be estimated. Our improvement of the Realized GARCH model is similar to using the score of a t-distribution which Harvey (2013) has advocated for conventional GARCH models. However, a full-fledged DCS/GAS structure is not needed in order to gain a robustness to large returns. We also make a similar adjustment to u_t, which measures the shocks to volatility, and substitute $\tilde{u}_t = u_t/(1 + u_t^2/d_u)$ for u_t in the GARCH equation. Specifically, we propose the following structure (henceforth M5):

$$r_t = \mu + \sqrt{h_t} z_t, \tag{6}$$

$$\log h_t = \omega + \beta \log h_{t-1} + \tau(\tilde{z}_{(1)t-1}) + \gamma \tilde{u}_{t-1} \tag{7}$$

$$\log x_t = \xi + \varphi \log h_t + \delta(z_t) + u_t, \tag{8}$$

approximately log-normal. Andersen, Bollerslev, Diebold and Ebens (2001) also find that the returns standardized by realized volatility are approximately normally distributed.

where $\tilde{z}_{(1)t} = z_t/(1 + z_t^2/d_{(1)z})$ and $\tilde{u}_t = u_t/(1 + u_t^2/d_u)$, with the leverage functions given by $\tau(\tilde{z}_{(1)}) = \tau_1\tilde{z}_{(1)} + \tau_2(\tilde{z}_{(1)}^2 - 1)$ and $\delta(z) = \delta_1 z + \delta_2(z^2 - 1)$. Additional variants of the robustified model were estimated and compared, see Appendix 6.1 and Appendix 6.2 for details. In our quasi maximum likelihood estimation we model z_t and u_t to be mutually and serially independent, with $z_t \sim \text{iid}N(0, 1)$ and $u_t \sim \text{iid}N(0, \sigma_u^2)$.

Given the structure of the model, the impact of jumps on the conditional volatility is twofold: it may come from the shocks on returns and/or the shocks on the realized measure of volatility. On the one hand, we measure today the impact of the shock suffered by the return series yesterday. Its effect on volatility is meant to be captured and controlled in the robust version of the model by the transformed innovation term $\tilde{z}_{(1)t} = z_t/(1 + z_t^2/d_{(1)z})$. On the other hand, the realized measure is very useful for modeling and forecasting future volatility and, therefore, it is used to explain the conditional volatility dynamics. To better account for its contribution, a second transformed innovation term, $\tilde{u}_t = u_t/(1 + u_t^2/d_u)$, is included in the GARCH equation. Furthermore, important shocks on the return series directly affect the proxy of volatility (through its construction) and the asymmetric response in the realized measure to these shocks is assessed by the transformed innovation term $\tilde{z}_{(2)t} = z_t/(1 + z_t^2/d_{(2)z})$ (that might be introduced in the measurement equation).

Creal et al. (2013) proved that it is reasonable to use the contribution of the score as an innovation in a time-varying parameter scheme, as the model makes use of the complete density structure of observations. Based on the same intuition, we use the score of a higher tailed distribution to transform the innovation term in order to dampen the influence of the outliers on the time-varying volatility term. For low values of the new parameters, observations that are considered outliers are far less influential. Therefore, this structure of the innovation terms controls the increase in volatility in case of large realizations of the returns.

Based on the Robust Realized GARCH model, volatility shocks become:

$$v_t = \tau(\widetilde{z}_{(1)t}) + \gamma\tilde{u}_t, \tag{9}$$

i.e., the sum of the news impact curve and the scaled shocks to the realized measure. The former is the new information about volatility changes captured by the return series and the latter is the new information captured by the realized measure.

3 Empirical Analysis

3.1 Data Description

We use high-frequency prices for the exchange traded fund, SPY, which closely tracks the S&P 500 index. The full sample spans the period from January 1, 1997 to December 31, 2009.

To estimate our models we use close-to-close returns. The realized measure of volatility captures only a fraction of the close-to-close volatility, since high-frequency data are available only from 9:30 am to 4 pm every trading day. The realized kernel is adopted as the realized measure, x_t, using the Parzen kernel function. This proxy of volatility is a more accurate estimator of the quadratic variation, dealing with issues such as jumps and other market microstructure noise. The realized kernel is implemented as in Barndorff-Nielsen et al. (2011) and guarantees a positive estimate, which is essential for the log-linear specification of the model. Before computing the return and realized measure series, intradaily data are first cleaned by following specific rules explained in great details in Barndorff-Nielsen et al. (2009). In addition, we remove days around Thanksgiving and Christmas in order to avoid outliers that would result from "quiet" days.

In order to quantify how much the stock price fluctuated on a day-to-day basis over a one-year period, we transform the conditional volatility estimates and the realized measure to an annualized scale. Annualized realized volatility is defined from the realized kernel estimates by:

$$\mathrm{Rvol}_t = \sqrt{250 \times \hat{c} \times \mathrm{RK}_t}, \qquad \hat{c} = \frac{\sum_t r_t^2}{\sum_t \mathrm{RK}_t},$$

while the annualized conditional variance corresponds to:

$$\mathrm{Cvol}_t = \sqrt{250 \times \mathrm{h}_t}.$$

3.2 Estimation Results

From the experience of GARCH-type models, the specification of the mean of returns typically does not make much difference. It is also true for our models. We estimated them with constant μ and find the estimated parameter $\hat{\mu}$ to be small and the implied $\hat{h}_t$ very close to those in the model with $\mu = 0$. Therefore, we have dropped the return constant in the estimated models.

Estimation results for the parsimonious form of the Robust Realized GARCH model (M5) (see Appendix 6.2) with the corresponding robust standard errors are presented below. To account for possible model misspecification, QMLE standard errors are computed using the sandwich estimator, see Bollerslev and Wooldridge (1992).

The following are the detailed results for the Robust Realized GARCH model based on SPY close-to-close returns for the full sample period of 1997 to 2009:

$$r_t = \sqrt{h_t} z_t$$

$$\log h_t = \underset{(0.004)}{0.013} + \underset{(0.004)}{0.967} \log h_{t-1} + \underset{(0.034)}{0.317} \tilde{u}_{t-1} - \underset{(0.009)}{0.149} \tilde{z}_{(1)t-1} + \underset{(0.007)}{0.051} (\tilde{z}_{(1)t-1}^2 - 1)$$

$$\log x_t = \underset{(0.037)}{-0.418} + \underset{(0.049)}{1.039} \log h_t - \underset{(0.009)}{0.133} z_t + \underset{(0.007)}{0.044} (z_t^2 - 1) + u_t$$

with $\hat{\sigma}_u^2 = \underset{(0.006)}{0.167}, \hat{d}_{(1)z} = 91.267, \hat{d}_u = 16.296.$

In order to focus on the recent financial crisis we also estimate the model using the recent sample that spans the period January 1, 2006 to December 31, 2009:

$$r_t = \sqrt{h_t} z_t$$

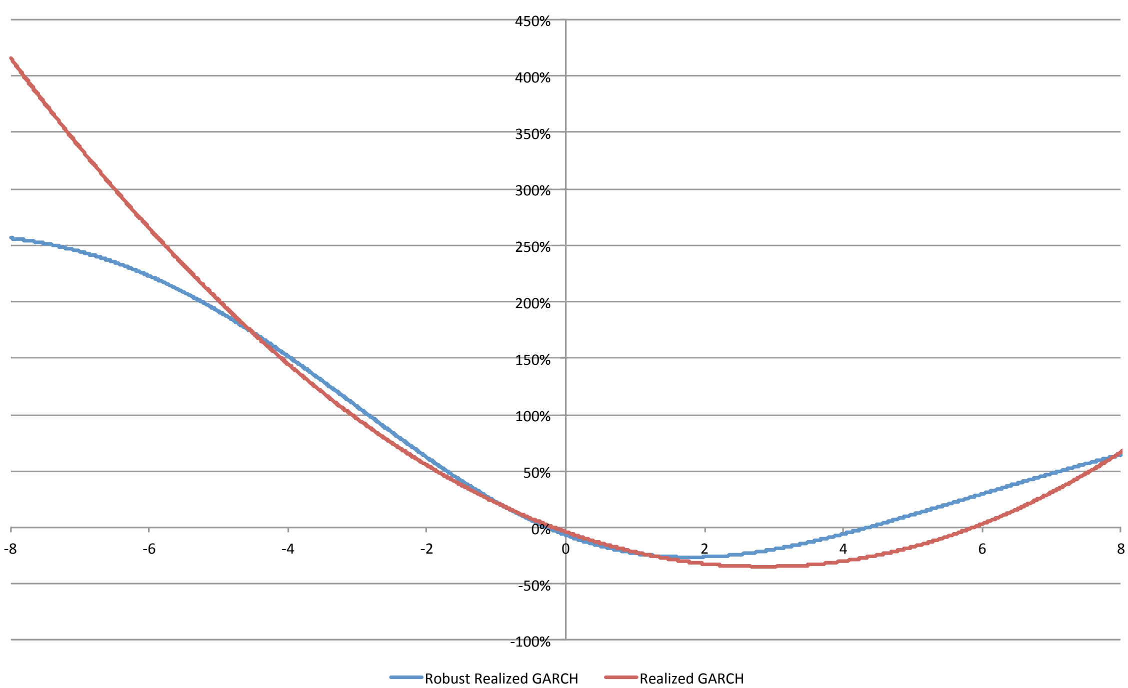
$$\log h_t = \underset{(0.009)}{0.015} + \underset{(0.006)}{0.969} \log h_{t-1} + \underset{(0.053)}{0.399} \tilde{u}_{t-1} - \underset{(0.016)}{0.177} \tilde{z}_{(1)t-1} + \underset{(0.012)}{0.052} (\tilde{z}^2_{(1)t-1} - 1)$$

$$\log x_t = \underset{(0.082)}{-0.532} + \underset{(0.071)}{1.019} \log h_t - \underset{(0.017)}{0.131} z_t + \underset{(0.009)}{0.036} (z_t^2 - 1) + u_t$$

with $\hat{\sigma}^2_u = \underset{(0.008)}{0.154}, \hat{d}_{(1)z} = 81.651, \hat{d}_u = 15.893.$

All of the parameters are statistically significant. The value of coefficient for $\tilde{u}_{t-1}$ ($\gamma = 0.399$) suggests that the realized measure provides informative signals in updating the conditional variance. A strong leverage effect is highlighted by the coefficient of $\widetilde{z}_{(1)t-1}$ in the GARCH equation and the coefficient of z_t in the measurement equation. Figure 2 displays the shape of the news impact curves when using the parameters estimated from the Realized GARCH model and its robustified version, respectively. It highlights the significant leverage effect for each of the two models. Bad news boost volatility while good news cause a reduction in it. In addition, this representation gives a first insight about the way the two models treat extreme shocks on volatility (*i.e.*, the Robust Realized GARCH model makes milder the impact of extreme shocks on volatility).

Figure 2: News Impact Curve



Note: This figure displays the news impact curves when using the parameters estimated from the Realized GARCH model and Robust Realized GARCH model, respectively.

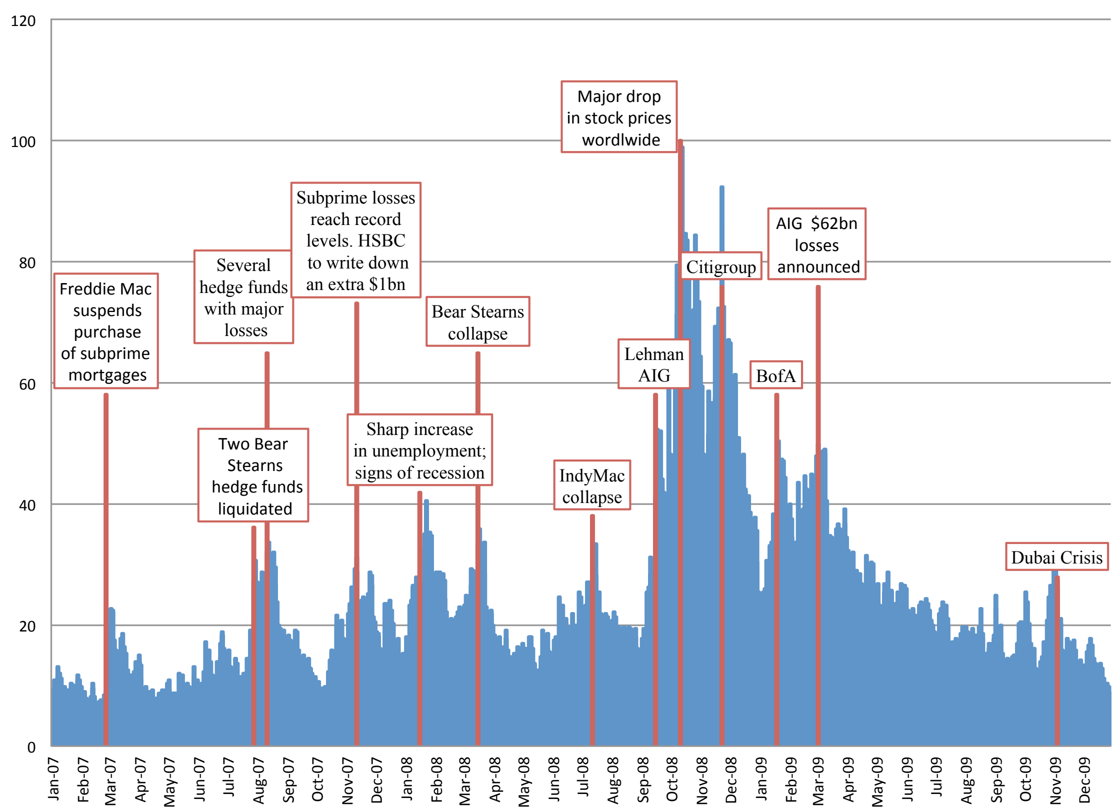
Moreover, we observe a high volatility persistence since the estimates of β in the GARCH equation are always close to 1. Besides, in the measurement equation we cannot reject the null hypothesis that the coefficient for $\log h_t$ equals unity, if computing and analyzing the $t-$statistics. This suggests that the realized measure is roughly proportional to the conditional variance. The new parameters defining the robustified model are estimated as $\hat{d}_{(1)z} = 81.651$ and $\hat{d}_u = 15.893$, respectively. It is important to note that the latter is normalized by the estimated variance of the idiosyncratic innovation in the measurement equation since $u_t \sim \text{iid}(0, \sigma_u^2)$.

3.3 Financial Events and High-frequency Data

This subsection addresses the identification of the main economic and/or financial events and their direct impact on volatility.

Figure 3 presents the annualized conditional volatility estimated by the Robust Realized GARCH model. It may be observed that during the recent global financial crisis (2007-2009) the main clusters of volatility overlap with significant economic and financial events. At the beginning of the financial crisis there exist clusters of volatility that correspond to major losses suffered by several hedge funds such as Bears Stearns, the announcement of a significant increase in the unemployment rate, etc. Subsequently, volatility exhibits peaks that could be associated with Lehman Brothers' collapse, the layoff plans announced by several companies (*i.e.*, General Motors, Wal-Mart Stores, UBS), the House of Representatives' decision to reject the $700 billion banking-rescue package etc.

Figure 3: Volatility by Robust Realized GARCH model

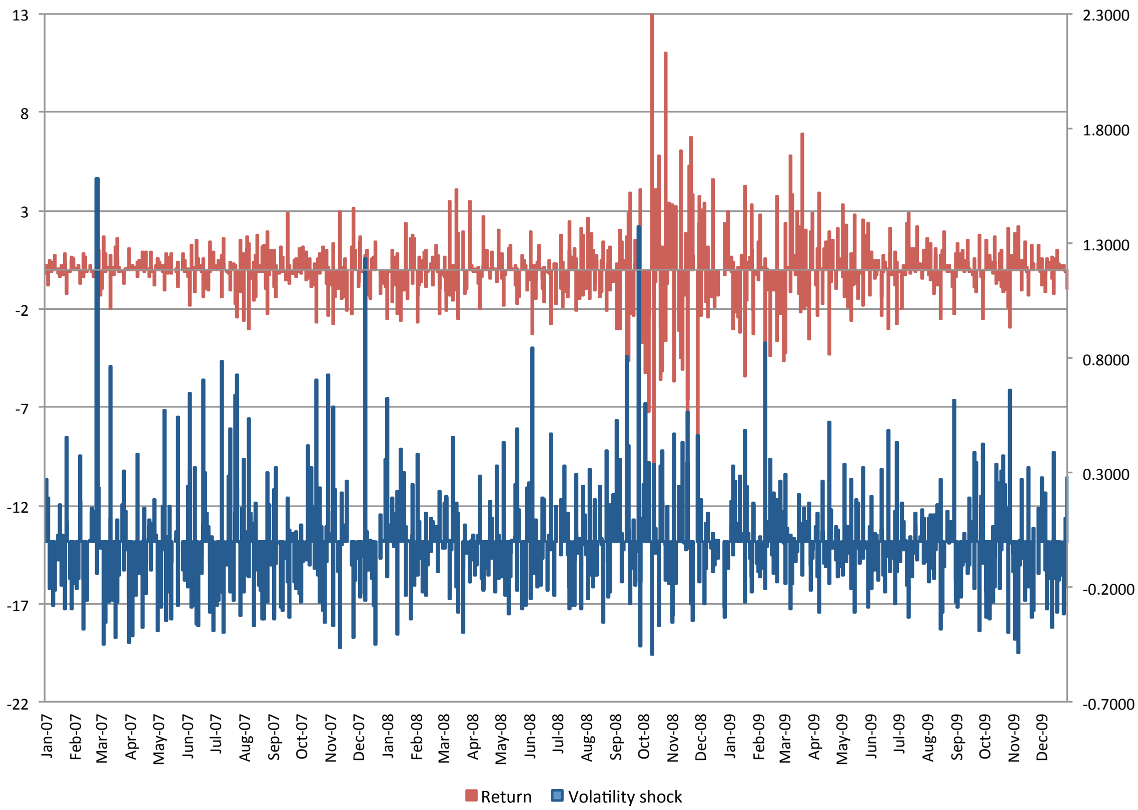


Note: This figure displays the conditional volatility, as estimated by the Robust Realized GARCH model, during the global financial crisis (2007-2009) with some of the major events.

Volatility shocks series, v_t (see Eq. 9), may be defined as a sum of the new information about volatility changes captured by the returns and the new information captured by the realized measure.

According to Figure 4, the largest positive volatility shock occurred at the beginning of the financial crisis, on February 27, 2007, when the markets were not that volatile. The return decreased to a negative abnormal value for that period. However, as shown in the returns graph, the biggest losses were recorded after September 2008, but, with some exceptions, the volatility shock series for this period is less significant than in the first part of the sample. This is consistent with the fact that the impact of financial/economic announcements may have a different intensity on volatility, conditional on the economy's shape at certain points in time. For instance, an announcement that occurs in a normal period may have a greater impact on volatility than if the same news would have occurred during a turmoil period. Consistent with theoretical beliefs, market reaction to news is greater if its surprise content is stronger.

Figure 4: Return and Volatility Shock series



Note: This figure displays the return and volatility shock series (Eq. 9) as estimated by the Robust Realized GARCH model during the period of 2007 to 2009.

In Table 1 it may be seen that the cluster of days with the largest positive volatility shocks is systematically the same across all seven models, albeit the order of the days slightly differs. Compared to the robust specification, the shocks modeled by the Realized GARCH are higher. The values become comparable when the intensity of shocks diminishes.

Figure 5 plots the realized measure of volatility and the conditional volatility estimated by the Realized GARCH model and M5 for the periods with the largest positive shocks. In the case of an extreme shock (February 27, 2007) M5 estimates the conditional volatility better than the Realized GARCH model. The latter overestimates the shock on volatility and the effect has a longer persistence in time. Moreover, the robust model performs well as regards the treatment of jumps. Given the financial markets state when this shock occurs (*i.e.*, the financial system does not face an alarming context), the robust model reduces the shock's impact on volatility and drives it quickly to the new level. This makes M5 more suited for situations when volatility exhibits "jumps" over a short period of time. Otherwise, the two models are comparable in terms of impact, evolution and persistence of shocks (see the part of the figure corresponding to the second and third largest shocks on volatility).

In what follows, we focus on the financial and/or economic announcements that induced the largest positive and negative shocks on volatility. Table 2 summarizes the ten largest positive volatility shocks as identified by M5, and the major events associated with them. The impact on volatility is calculated as $(\exp(v_t) - 1) * 100$, where v_t accounts for the volatility shock. This impact ranges from 107% to 386%.

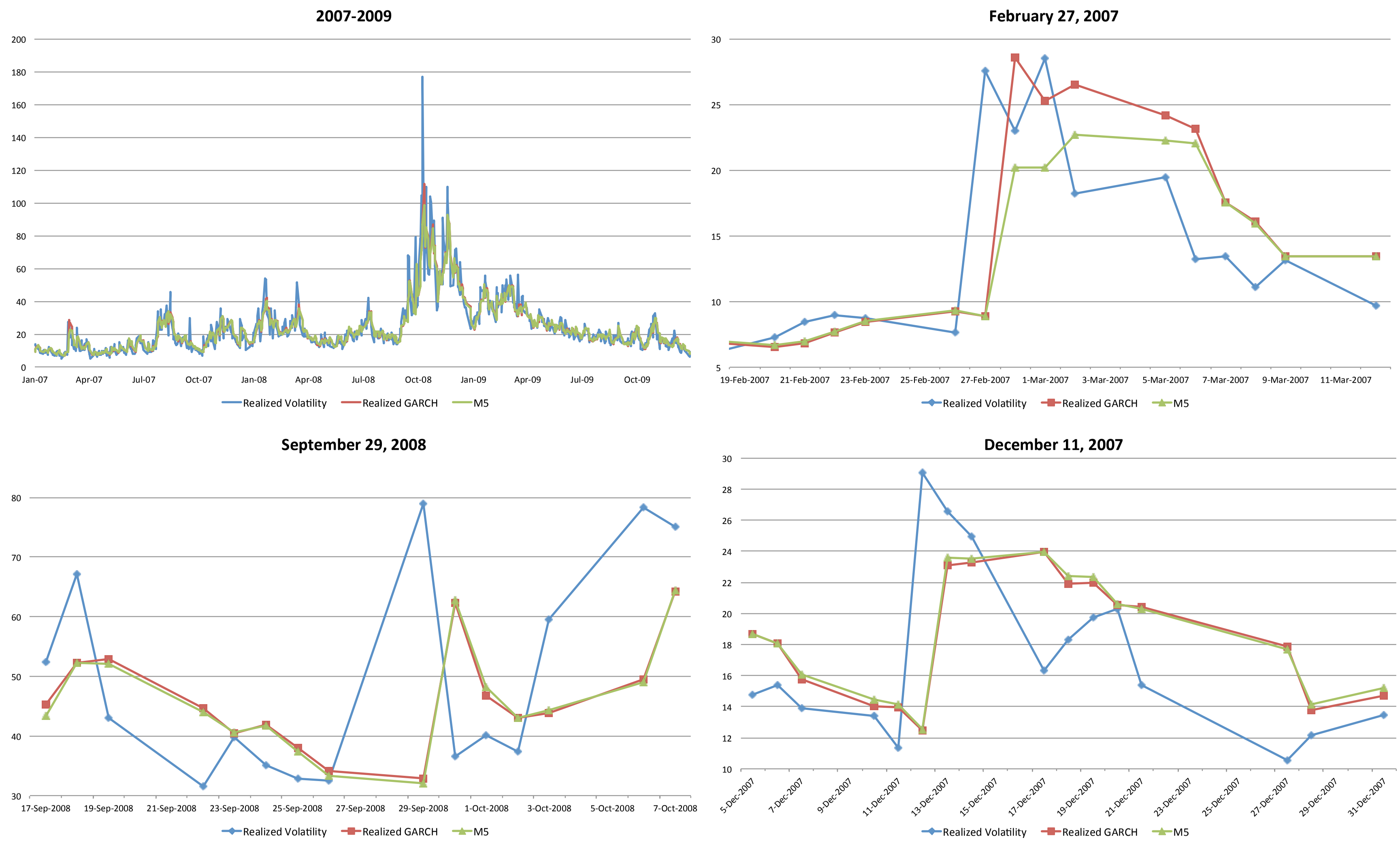
Table 3 reports the five largest negative volatility shocks and the major events associated with them. Their impact on volatility is lower than the impact of the positive shocks, supporting the leverage effect view (*i.e.*, bad news have a more disturbing effect on volatility than good news).

Table 1: The ten largest positive volatility shocks

Realized GARCH		M1		M2		M3		M4		M5		Robust Realized GARCH	
Date	Vol shock	Date	Vol shock	Date	Vol shock	Date	Vol shock	Date	Vol shock	Date	Vol shock	Date	Vol shock
20070227	**2.300**	**20070227**	**2.300**	**20070227**	**2.190**	**20070227**	**1.700**	**20070227**	**1.680**	**20070227**	**1.580**	**20070227**	**1.600**
20080929	**1.310**	**20080929**	**1.310**	**20080929**	**1.400**	**20080929**	**1.400**	**20080929**	**1.400**	**20080929**	**1.370**	**20080929**	**1.370**
20071211	**1.210**	**20071211**	**1.190**	**20071211**	**1.280**	**20071211**	**1.280**	**20071211**	**1.270**	**20071211**	**1.230**	**20071211**	**1.250**
20080606	0.790	20080606	0.790	20080606	0.850	20090210	0.870	20090210	0.870	20090210	0.870	20090210	0.880
20090210	0.780	20090210	0.780	20090210	0.830	20080606	0.860	20080606	0.860	20080606	0.850	20080606	0.850
20070726	0.730	20070726	0.710	20080915	0.760	20080915	0.810	20080915	0.800	20080915	0.810	20080915	0.810
20080915	0.700	20070710	0.710	20070726	0.760	20070710	0.780	20070710	0.770	20070710	0.780	20070710	0.800
20070710	0.700	20080915	0.710	20070710	0.750	20070726	0.770	20070726	0.760	20070313	0.760	20070313	0.770
20070313	0.660	20070313	0.670	20070313	0.720	20070313	0.760	20070313	0.750	20071101	0.730	20070726	0.740
20071101	0.640	20071101	0.650	20071101	0.690	20071101	0.720	20071101	0.710	20070726	0.730	20071101	0.740

Note: This table presents the ten largest positive volatility shocks computed as $v_t = \tau(\tilde{z}_{(1)t}) + \gamma\tilde{u}_t$, along with the corresponding dates of occurrence.

Figure 5: Evolution of the realized measure of volatility and the conditional volatility



Note: This figure presents the evolution of the realized measure of volatility and the conditional volatility for the period of 2007 to 2009, as well as for the periods embedding the three largest positive shocks on volatility, *i.e.*, February 27, 2007, September 29, 2008 and December 11, 2007. The blue line represents the realized measure of volatility, while the red and green lines correspond to the conditional volatility from the Realized GARCH and Robust Realized GARCH (M5) models, respectively.

Table 2: The ten largest positive volatility shocks and associated events

No	**Date**	**Impact on volatility (in %)**	r_t (%)	**Events**
1	20070227	386	-3.98	* China stock market dropped by 8.8%. * Freddie Mac announced to tighten standards on subprime loans. * NYSE trading interrupted due to a computer glitch (3 pm). * News of a suicide bombing attack at the entrance to the main U.S. military base in Afghanistan during a visit by Dick Cheney.
2	20080929	294	-8.16	* The House of Representatives rejected the $700 billion banking-rescue package. * Wachovia announced the selling of the banking operation of Citibank. * The crisis has spread to the European financial system (*e.g.*, the Icelandic government nationalizes the bank Glitnir with the purchase of a 75% stake for €600 million).
3	20071211	243	-2.78	* Fed cut the federal funds rate by 0.25% to 4.25%. * Negative news for Freddie Mac, Washington Mutual, Fannie Mae, etc.
4	20090210	139	-3.24	* Obama administration unveiled the new rescue package but the investment community worried that the rescue plan would prove inadequate in the face of a recession. * Large layoffs plans are announced by several companies (*i.e.*, General Motors, Wal-Mart Stores, UBS).
5	20080606	133	-4.69	* Lehman Brothers announced the plans to raise $5 -6 billion in fresh capital as it disclosed a large second-quarter loss. * MBIA and Ambac were downgraded two notches from AAA to AA. * Unexpectedly high May unemployment rate is announced (5.5% from 5.0% in April).
6	20080915	125	-4.87	* Lehman Brothers Holdings Inc filed for Chapter 11 bankruptcy protection. * Merrill Lynch agreed to be sold to Bank of America.
7	20070710	119	-1.43	* Standard and Poor's Rating Services put 612 securities on "CreditWatch negative" because of high delinquency and foreclosure rates. Moody's Investors Service downgraded 399 securities and placed an additional 32 securities on review for possible downgrade.
8	20070313	114	-1.96	* Worries about subprime lending. * The dollar tumbled versus other major currencies.
9	20071101	107	-2.37	* Downgrade of Citigroup. * Credit Suisse reported a 31 percent drop in profits. * Exxon Mobil reported a bigger-than-expected drop in quarterly earnings. * Moody's, Standard & Poor's and Fitch put an estimated $70 billion worth of collateralized debt obligations on review for downgrading. * Economic reports on personal income and spending, manufacturing, foreclosure filings shifted the attention of investors.
10	20070726	107	-2.39	* Wells Fargo & Co. announced that it will stop making subprime mortgages through brokers amid escalating late payments and defaults. * NYSE imposed trading curbs to slow down the market in the event of a big move. * Homebuilders posted huge losses (new house sales tumbled 6.6%).

Table 3: The five largest negative volatility shocks and associated events

No	**Date**	**Impact on volatility (in %)**	r_t**(%)**	**Events**
1	20081013	-39	13.56	∗ Global plans for governments to rescue banks through direct capital injections (*e.g.*, buying soured mortgage assets from banks and stock in a number of financial institutions, injections of capital, etc.). ∗ The European Central Bank attempts to revive credit market by weekly injections of unlimited euro funds at an interest rate of 3.75%. ∗ The U.S. central bank would provide unlimited dollars to the European Central Bank, Bank of England and Swiss National Bank, allowing them to relieve pressures on commercial banks across their regions.
2	20091109	-39	2.25	∗ Finance ministers of the G-20 met over the weekend and pledged to keep economic stimulus in place.
3	20071113	-37	3.00	∗ Positive comments came out of the Merrill Lynch conference (*e.g.*, Goldman Sachs outlook, etc.). ∗ Wal-Mart Stores reported higher third-quarter earnings than topped estimates. ∗ Retreat in oil prices from near record levels. ∗ Extra boost of stocks in the afternoon from the home sales index that rose 0.2% versus forecasts for a drop of 2.5%.
4	20080930	-36	4.06	∗ Stocks rallied one day after the failure of the Congress to pass a $700 billion financial bailout plan.
5	20071221	-36	1.43	∗ The Federal Reserve announced that it lent $20 billion to banks at an interest rate of 4.67% in order to support the credit markets. ∗ The "Super SIV" rescue fund was canceled as the consortium claimed that "[it] is not needed at this time". ∗ Encouraging economic news about personal income and spending.

4 News Related to the Largest Volatility Shocks

The next section describes the events recorded in the three most volatile days, disseminating the causes of the shocks by employing an econometric technique which allows shock identification on a high-frequency basis.

4.1 February 27, 2007

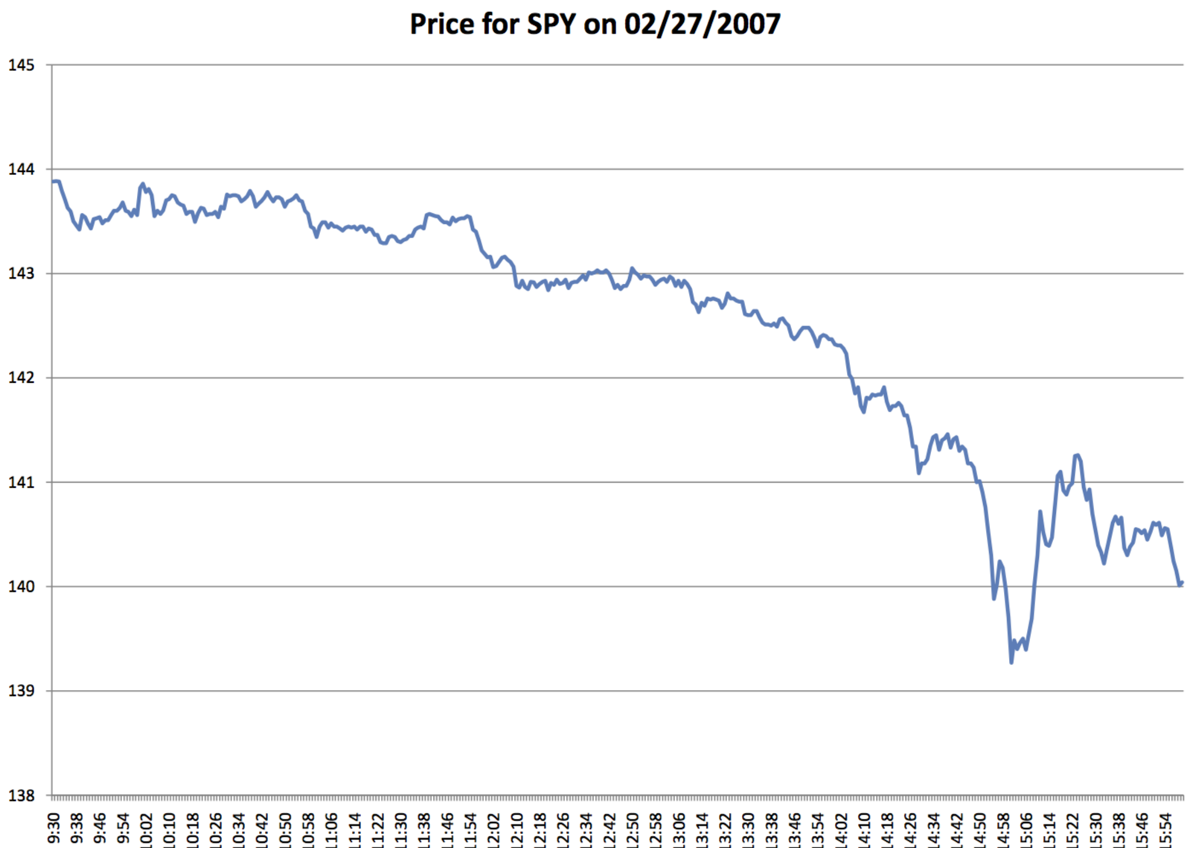
February 27, 2007 corresponds to the largest positive volatility shock, although not akin to a turbulent financial period. On that day the decade's biggest drop in the Chinese stock market was recorded by the Shanghai Composite Index (-8.5%), decline seen as a symptom of broader concerns on global valuations. Fearing state intervention (including new trading taxes) to temper the market's exuberance, investors promptly left de Chinese soaring market. As an effect of the Chinese drop, the Dow Jones Industrial Average index began its day under pressure.

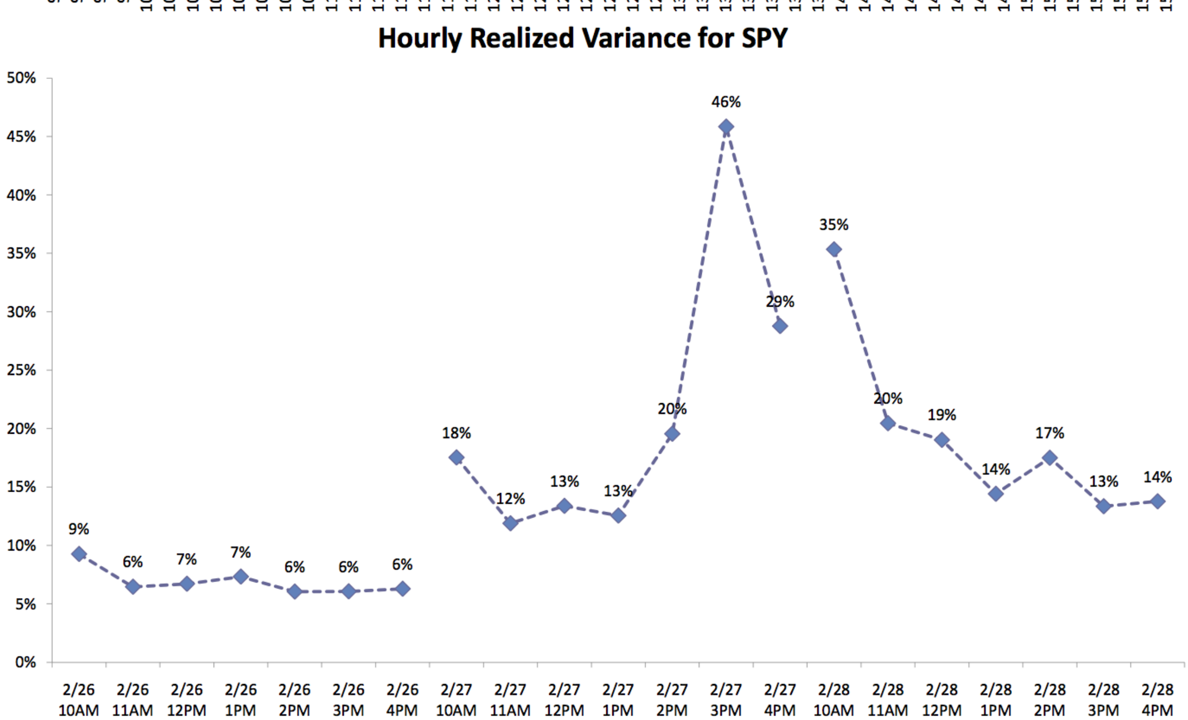
Additional factors such as the news of a suicide bombing attack at the entrance to the main U.S. military base in Afghanistan during a visit by Dick Cheney, and worries of a possible recession fueled by a statement of the former chairman of the Federal Reserve, Alain Greenspan, caused the downgrade in financial markets. Freddy Mac also announced that the company intended to tighten standards on subprime loans by stopping to purchase loans granted to subprime borrowers or those with weak credit records. Accordingly, on September 1, 2007 Freddie would have stopped buying subprime loans that were likely to lead to "payment shocks" when rates would have been reset after two or three years.

By using an econometric model that benefits from the information available at high-frequency level we better understand event-shock results. Figure 6 displays the intraday price pattern of SPY on February 27, 2007, as well as the hourly realized volatility for the same day and for the previous and following day.

The shock occurred at around 3 pm, which coincides with a computer glitch in the trading system that occurred just before 3 pm. According to the Dow Jones spokeswoman: "around 2:00 pm [on that day] the market's extraordinary heavy trading volume caused a delay in the Dow Jones data systems. [...] and as we identified the problem we decided to switch to a back-up system and the result was a rapid catch-up in the published value of the Dow Jones Industrial Average." The back-up system was activated around 3:00 pm and at 3:02 pm the Dow downgraded by 160 points and continued its depreciation throughout the afternoon. The Dow Jones Industrial average index fell by 546 points in the afternoon. This drop in US stock market was considered as the steepest since September 11, 2001, as it determined a flight to safety in bond markets of previously risky investors. The decrease of the price (between 2:50 pm and 3:05 pm) is readily identifiable in Figure 6.

Figure 6: Intradaily pattern of the price series and the realized measure on February 27, 2007





The impact of the other events which occurred that day is also important. Nevertheless, their effect on volatility would have been overestimated without knowing the exact time of the shock. Under these circumstances, the intraday analysis helps us to determine the main cause leading to this significant increase in volatility.

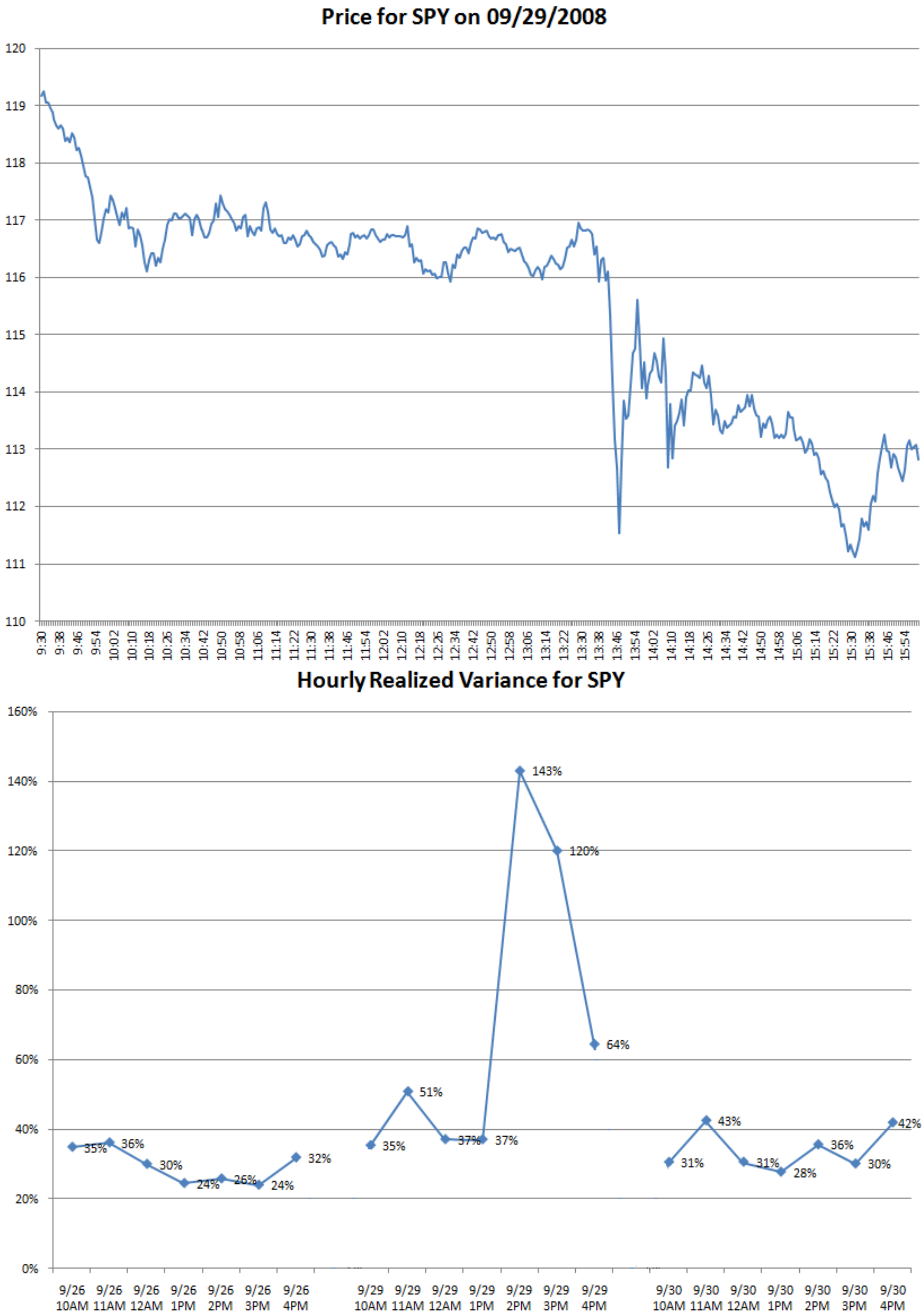
4.2 September 29, 2008

The second largest shock on volatility occurred on September 29, 2008. As shown in Figure 7, prices plunged significantly in the afternoon between 1:30 pm and 1:45 pm. At that time, the House of Representatives rejected (with a 228-205 vote) Bush Administration's $700 billion banking-rescue package, hence placing the stock market into a tailspin and creating worries about a prolonged recession. The bailout was designed to revive the financial institutions' lending by removing from the market the toxic mortgage-backed securities and other holdings that lenders feared they could cause borrowers to default. Referring to the House's vote, Drew Kanaly, chairman and CEO of Kanaly Trust Company said that "the stock market was definitely taken by surprise." As a consequence, the Standard & Poor's 500 index depreciated by 8.8% on that day, its seventh worst day ever on a percentage basis and the biggest one-day percentage drop since the crash of '87, when it was devalued by 20.5%.

Other news also contributed to the volatility spike recorded on September 29, 2008. Wachovia announced the selling of the banking operation of Citigroup for $2.2 billion in an all-stock deal. This led Wachovia shares to a loss of 81% in the afternoon, while Citigroup fell almost 12%. Moreover, the British government nationalized the mortgage lender Bradford & Bingley PLC and some European banks collapsed. The German commercial property lender Hypo Real Estate Group opted for a government-facilitate credit line because of difficulties caused by the international credit-market turmoil. The government of Iceland took control of Glitnir, the country's third largest bank, to avoid its likely collapse. Moreover, over the weekend, Fortis was partially nationalized, receiving a €11.2 billion capital injection from Netherlands, Belgium and Luxembourg.

In addition, another objective cause of the state of volatility is related to the anxiety provoked by a large number of last-minute activities, since this date coincided also with the end of the third quarter.

Figure 7: Intradaily pattern of the price series and the realized measure on September 29, 2008

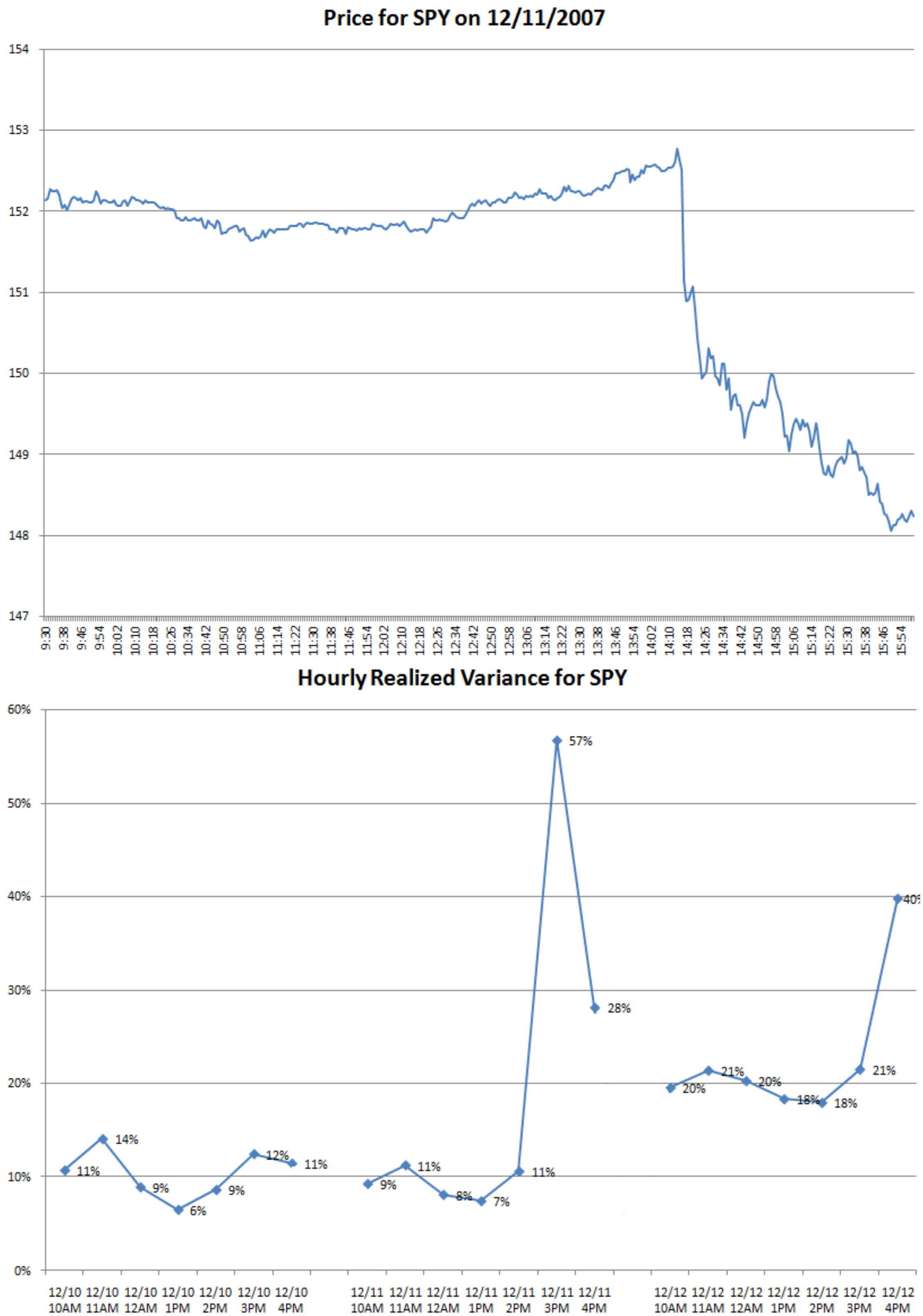


4.3 December 11, 2007

The S&P 500 index lost 2.5% on December 11, 2007. In order to promote moderate growth over time, Fed cut the federal funds rate charged on overnight loans between banks by 0.25 percentage point to 4.25%. The meeting was held at 8 am and the news were released at around 11:34 am. The market initially cheered the anticipated move, but the joy quickly retreated on the disappointment that the Fed did not cut interest rates more. Markets may have also anticipated the step taken the next day, on December 12th, when central banks from five major currency areas (the Federal Reserve, European Central Bank, Bank of England, Bank of Canada and Swiss National Bank) announced coordinated measures designed to stem the credit crunch and address pressures in short-term funding markets by making cash more readily available to banks. Bank of Japan and Bank of Sweden also expressed support for these initiatives. As such, the same day of December 12, the Fed announced the holding in the eight-day period to follow of two auctions of $20 billion each, in special one-month loans. Supplementary, the Fed would create a new liquidity facility that would allow banks to bid for loans by presenting as collateral a broad range of assets, including securities related to the housing sector. Moreover, special currency swap arrangements were planned to alleviate the offshore pressure on overnight federal funds rate and the longer term interbank market that would permit European banks to get dollars from their own central banks instead. Similarly, the European Central Bank and the Swiss National Bank announced their entrance into swap arrangements with the Fed in order to provide up to $24 billion for distribution to banks in Europe in need of dollar funds. Nevertheless, the Bank of England and the Bank of Canada announced the enforcement of some sweeping changes to their regulations on collateral in order to permit financial institutions to pledge a broader range of securities in exchange for funds.

In addition, more negative news, coming from some notable components of Dow, surprised the financial markets (*e.g.*, Washington Mutual announced that it was cutting both dividends and more than 3,000 jobs (December 10), Freddie Mac's chief executive presumed to lose $5.5 billion to $7.5 billion over the next few years, etc.) (December 11, 10:20 am). According to the timing of the events, the volatility shocks were triggered both by the Fed rate cut by 0.25 percentage points and the Freddie Mac's announcement on presumed losses.

Figure 8: Intradaily pattern of the price series and the realized measure on December 11, 2007



5 Conclusion

In this chapter we accomplish two main objectives. In a first time we propose a new and robust version of the Realized GARCH approach that better models volatility by accounting for both the asymmetry and the jumps effects. To this end, we rely on the Realized GARCH framework proposed by Hansen and Huang (2012), that involves a slightly different approach to the joint modeling of returns and realized volatility measures. The econometric novelty of the paper is the improvement of this model based on the intuition in Creal et al. (2011, 2013) and Harvey (2013), who demonstrate that it is reasonable to use the contribution of the score function as an innovation in a time-varying parameter scheme. Based on the same idea, we use the score of a higher tailed distribution to transform the innovation term in order to dampen the influence of the outliers on the time-varying volatility term. Therefore, the observations that are considered outliers are far less influential.

Second, we define the volatility shock series that incorporates both the new information about volatility changes captured by the returns and the new information captured by the realized measure of volatility. Using the largest positive and negative shocks on volatility we identify the main events (*e.g.*, financial, economic, governmental, social, etc.) that could have induced these shocks. We observe that the largest positive shock on volatility occurred at the beginning of the recent financial crisis, on February 27, 2007, when the markets were not that volatile. From the intraday data we observe a perfect match between the volatility shock and the occurrence of a computer glitch in the trading system (just before 3 pm). We do not have to neglect the importance of the other events occurred that day, such as the decade's biggest drop in the Chinese stock market, the worries on a possible recession fueled by a statement of the former chairman of the Federal Reserve, or the Freddy Mac's announcement about tightening standards on subprime loans. However, their impact would have been overestimated if we had not proceeded to a closer intradaily analysis. The other biggest shocks on volatility were fueled by governmental decisions, economic and financial news or social events.

6 Appendix

6.1 Appendix A: Variants of Robustified Realized GARCH

Realized GARCH:

$$
\begin{aligned}
r_t &= \mu + \sqrt{h_t} z_t, && (10)\\
\log h_t &= \omega + \beta \log h_{t-1} + \tau(\tilde{z}_{t-1}) + \gamma \tilde{u}_{t-1} && (11)\\
\log x_t &= \xi + \varphi \log h_t + \delta(\tilde{z}_t) + u_t && (12)
\end{aligned}
$$

where:

$\tau(z) = \tau_1 \tilde{z} + \tau_2(\tilde{z}^2 - 1)$

$\delta(z) = \delta_1 \tilde{z} + \delta_2(\tilde{z}^2 - 1)$

$\tilde{z}_t = z_t$ and $\tilde{u_t} = u_t$

with z_t and u_t assumed to be mutually and serially independent, with $z_t \sim \text{iid}(0,1)$ and $u_t \sim \text{iid}(0, \sigma_u^2)$. We adopt the Gaussian specification in our quasi likelihood analysis.

Model 1 (M1):

$$
\begin{aligned}
r_t &= \mu + \sqrt{h_t} z_t && (13)\\
\log h_t &= \omega + \beta \log h_{t-1} + \tau(z_{t-1}) + \gamma \tilde{u}_{t-1} && (14)\\
\log x_t &= \xi + \varphi \log h_t + \delta(z_t) + u_t && (15)
\end{aligned}
$$

where

$\tau(z) = \tau_1 z + \tau_2(z^2 - 1)$

$\delta(z) = \delta_1 z + \delta_2(z^2 - 1)$

$\tilde{u_t} = u_t/(1 + u_t^2/d_u)$

Model 2 (M2):

$$
\begin{aligned}
r_t &= \mu + \sqrt{h_t} z_t && (16)\\
\log h_t &= \omega + \beta \log h_{t-1} + \tau(\tilde{z}_{t-1}) + \gamma u_{t-1} && (17)\\
\log x_t &= \xi + \varphi \log h_t + \delta(\tilde{z}_t) + u_t && (18)
\end{aligned}
$$

where:

$\tau(\tilde{z}) = \tau_1 \tilde{z} + \tau_2(\tilde{z}^2 - 1)$

$\delta(\tilde{z}) = \delta_1 \tilde{z} + \delta_2(\tilde{z}^2 - 1)$

$\tilde{z}_t = z_t/(1 + z_t^2/d)$

Model 3 (M3):

$$r_t = \mu + \sqrt{h_t} z_t \tag{19}$$

$$\log h_t = \omega + \beta \log h_{t-1} + \tau(\tilde{z}_{(1)t-1}) + \gamma u_{t-1} \tag{20}$$

$$\log x_t = \xi + \varphi \log h_t + \delta(\tilde{z}_{(2)t}) + u_t \tag{21}$$

where:

$\tau(\tilde{z}_{(1)}) = \tau_1 \tilde{z}_{(1)} + \tau_2(\tilde{z}_{(1)}^2 - 1)$

$\delta(\tilde{z}_{(2)}) = \delta_1 \tilde{z}_{(2)} + \delta_2(\tilde{z}_{(2)}^2 - 1)$

$\tilde{z}_{(1)t} = z_t/(1 + z_t^2/d_{(1)z})$

$\tilde{z}_{(2)t} = z_t/(1 + z_t^2/d_{(2)z})$

Model 4 (M4):

$$r_t = \mu + \sqrt{h_t} z_t \tag{22}$$

$$\log h_t = \omega + \beta \log h_{t-1} + \tau(\tilde{z}_{(1)t-1}) + \gamma u_{t-1} \tag{23}$$

$$\log x_t = \xi + \varphi \log h_t + \delta(z_t) + u_t \tag{24}$$

where:

$\tau(\tilde{z}_{(1)}) = \tau_1 \tilde{z}_{(1)} + \tau_2(\tilde{z}_{(1)}^2 - 1)$

$\delta(z) = \delta_1 z + \delta_2(z^2 - 1)$

$\tilde{z}_{(1)t} = z_t/(1 + z_t^2/d_{(1)z})$

Model 5 (M5):

$$r_t = \mu + \sqrt{h_t} z_t \tag{25}$$

$$\log h_t = \omega + \beta \log h_{t-1} + \tau(\tilde{z}_{(1)t-1}) + \gamma \tilde{u}_{t-1} \tag{26}$$

$$\log x_t = \xi + \varphi \log h_t + \delta(z_t) + u_t \tag{27}$$

where:

$\tau(\tilde{z}_{(1)}) = \tau_1 \tilde{z}_{(1)} + \tau_2(\tilde{z}_{(1)}^2 - 1)$

$\delta(z) = \delta_1 z + \delta_2(z^2 - 1)$

$\tilde{z}_{(1)t} = z_t/(1 + z_t^2/d_{(1)z})$

$\tilde{u}_t = u_t/(1 + u_t^2/d_u)$

Robustified Realized GARCH:

$$r_t = \mu + \sqrt{h_t} z_t \tag{28}$$

$$\log h_t = \omega + \beta \log h_{t-1} + \tau(\tilde{z}_{(1)t-1}) + \gamma \tilde{u}_{t-1} \tag{29}$$

$$\log x_t = \xi + \varphi \log h_t + \delta(\tilde{z}_{(2)t}) + u_t \tag{30}$$

where:

$\tau(\tilde{z}_{(1)}) = \tau_1 \tilde{z}_{(1)} + \tau_2(\tilde{z}^2_{(1)} - 1)$

$\delta(\tilde{z}_{(2)}) = \delta_1 \tilde{z}_{(2)} + \delta_2(\tilde{z}^2_{(2)} - 1)$

$\tilde{z}_{(1)t} = z_t/(1 + z_t^2/d_{(1)z})$

$\tilde{z}_{(2)t} = z_t/(1 + z_t^2/d_{(2)z})$

$\tilde{u}_t = u_t/(1 + u_t^2/d_u)$

6.2 Appendix B: Variants of Robustified Realized GARCH (Estimation)

As previously noted, the Robust Realized GARCH model controls the impact of jumps on volatility and the realized measure. Moreover, its general form nests many simplified specifications. For instance, we can consider a specific form of the robust version that only deals with the impact of jumps on the conditional volatility (*i.e.*, $\tilde{z}_{(2)t} = z_t$), or another form that treats identically the impact of jumps in the return series on the conditional volatility and the realized measure (*i.e.*, $d_{(1)z} = d_{(2)z}$), and so on.

In this section we shed light on the Robust Realized GARCH structure (both general and simplified forms) and subsequently compare its performances in terms of empirical fit with those of the standard Realized GARCH. To this end, we estimate the general form of the robust version of the model previously presented, five other different models that are nested into this general specification, as well as the standard Realized GARCH, for the period of 2006 to 2009. Table 4 presents the parameter estimates, as well as the value of the log-likelihood function of the Realized GARCH model, the robustified Realized GARCH model and the five extensions denoted M1-M5.

The best model in terms of log-likelihood is, obviously, the Robust Realized GARCH, albeit it is closely followed by M5 (the simplified form that adjusts the impact of jumps only on the conditional volatility). However, when computing the log-likelihood Ratio (LR) test we observe that the difference between these two models is not statistically significant. Moreover, the new parameter of the transformed innovation term that appears into the measurement equation of the general robust model is quite large, allowing to presume $\tilde{z}_{(2)t} = z_t$. It means that we can control for the asymmetry and the impact of the outliers by changing the innovations structure only in the conditional volatility formula (*i.e.*, the GARCH equation). The estimated parameter associated with the number of degrees of freedom appearing into the transformed innovation term $\tilde{u}_t$ ($d_u = 15.8934$) is lower than the one associated with $\tilde{z}_{(1)t}$ ($d_{(1)z} = 81.6507$), which suggests that the influence of the outliers coming from the realized volatility series is even more rigorously controlled. In addition, M5 is 6 units better than the classical Realized GARCH specification, highlighting the statistical benefits of incorporating the score function into the definition of the innovation terms corresponding to the GARCH equation.

To statistically assess the performance of the new model in terms of the in-sample goodness of fit, numerous tests are available. A standard one is the LR test previously mentioned, which is based on a pairwise comparison meant to compare each two candidate models. However, since the number of competing volatility models is important, we rather focus on multiple comparison-based tests in order to obtain a joint confidence interval for all possible pairwise comparisons. We apply here the model confidence set (MCS) test of Hansen et al. (2011) by using the Kullback-Leibler information criterion (KLIC).[3] This is equivalent to providing rankings of models in terms of their expected value of the quasi-log-likelihood function. Following Hansen et al. (2011), we set the significance level at 25% and use 10,000 block bootstrap resamples (with block length of five observation) to obtain the distribution under the null of equal empirical fit.

[3]The MCS test that can be used for specific research topics, such as comparing the empirical fit of regression models, or the out-of-sample forecasts accuracy. This test identifies among an universe of competing models, the subset of models that are equivalent in terms of predictive abilities or in-sample empirical fit, but which outperform all the other models at a confidence level α.

Table 4: Parameter estimates for Realized GARCH model, Robust Realized GARCH model and its extensions

Parameter	**Realized GARCH**	**M1**	**M2**	**M3**	**M4**	**M5**	**Robust Realized GARCH**
$d_{(1)z}$			148.7753	77.9859	95.6041	81.6507	67.4762
$d_{(2)z}$			148.7753	290.4173			290.4391
d_u		29.7636				15.8934	15.5890
h_0	0.7974	0.8112	0.7820	0.7920	0.8022	0.8218	0.8139
ω	0.0058	0.0066	0.0098	0.0140	0.0123	0.0146	0.0165
β	0.9719	0.9722	0.9706	0.9685	0.9685	0.9689	0.9689
γ	0.3678	0.3994	0.3643	0.3565	0.3510	0.3990	0.4062
τ_1	-0.1713	-0.1707	-0.1767	-0.1794	-0.1761	-0.1768	-0.1804
τ_2	0.0249	0.0250	0.0429	0.0547	0.0489	0.0517	0.0573
ξ	-0.5182	-0.5196	-0.5160	-0.5254	-0.5301	-0.5320	-0.5275
φ	1.0058	1.0050	0.9932	1.0079	1.0211	1.0193	1.0069
δ_1	-0.1277	-0.1291	-0.1329	-0.1308	-0.1288	-0.1315	-0.1338
δ_2	0.0365	0.0359	0.0515	0.0461	0.0376	0.0363	0.0445
σ_u^2	0.1574	0.1570	0.1559	0.1546	0.1548	0.1539	0.1537
AIC	4026.11	4026.28	4024.82	4020.29	4019.75	**4017.94**	4018.56
BIC	4080.02	4085.08	4088.53	4084.00	**4078.55**	4081.65	4087.17
logL	-2002.05	-2001.14	-1999.41	-1997.14	-1997.8729	**-1995.97**	-1995.28

Note: The variants of the Robust Realized EGARCH model have the following characteristics:

- M1: $\tilde{z}_{(1)t} = z_t$; $\tilde{z}_{(2)t} = z_t$; $\tilde{u}_t = u_t/(1 + u_t^2/d_u)$
- M2: $\tilde{z}_{(1)t} = \tilde{z}_{(2)t} = z_t/(1 + z_t^2/d)$; $\tilde{u}_t = u_t$
- M3: $\tilde{z}_{(1)t} = z_t/(1 + z_t^2/d_{(1)z})$; $\tilde{z}_{(2)t} = z_t/(1 + z_t^2/d_{(2)z})$; ; $\tilde{u}_t = u_t$
- M4: $\tilde{z}_{(1)t} = z_t/(1 + z_t^2/d_{(1)z})$; $\tilde{z}_{(2)t} = z_t$; $\tilde{u}_t = u_t$
- M5: $\tilde{z}_{(1)t} = z_t/(1 + z_t^2/d_{(1)z})$; $\tilde{z}_{(2)t} = z_t$; $\tilde{u}_t = u_t/(1 + u_t^2/d_u)$

Table 5 reports the realizations of the empirical KLIC with the corresponding *p*-values of the MCS test for the seven competing models. We notice that, for a significance level of 25%, the cluster of superior models in terms of in-sample empirical fit is mainly dominated by the general form of the Robust Realized GARCH and four of its simplified forms (M2, M3, M4 and M5). As expected, the MCS procedure does not select the standard Realized GARCH model. These results confirm our intuition that the Robust Realized GARCH model statistically outperforms the standard version in terms of the in-sample goodness of fit.[4]

[4]We do not perform an out-of-sample analysis as this task is not so decisive for the rest of the analysis.

Table 5: KLICs and MCS p-values

Model	**KLIC**	**MCS p-value**
Realized GARCH	4.0323	0.2014
M1	4.0304	0.2233
M2	4.0270	0.3960
M3	4.0224	0.6006
M4	4.0239	0.5100
M5	4.0201	0.6006
Robust Realized GARCH	4.0186	1

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