

Volatility contagion: new evidence from market pricing of the volatility risk

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Abstract

This paper proposes a novel test for the volatility contagion on the equity markets. I decompose variance risk premia into their tail and non-tail risk components for three major stock indices and I analyse their cross market correlations. I find that tail-risk premia exhibit higher correlations than the non-tail risk premia, implying the existence of volatility contagion. This result holds, even when allowing for time varying correlations. Moreover I document that tail-premia constitute a large portion of the overall premia, highlighting even more the importance of tail-risks.

Unlike the existing literature, my approach to testing the existence of volatility contagion does not rely on short periods of financial distress. The decomposition allows to gauge the tail risk premia also in tranquil times.

Key words: Financial Contagion, Variance Risk Premium, Tail-risk, Equity co-movement, DCC model.

JEL classification: C58; F36; G12; G13; G15

1 Introduction

The Great Recession highlighted once more the importance of market contagion, both for policy makers and for the financial industry. Events following the collapse of the Lehman Brothers especially underlined the issue of market uncertainty contagion.¹ This type of contagion might be present across different asset classes as well as across different markets. Surprisingly there is no consensus whether contagion actually exists or not.²

Throughout this study, following Forbes and Rigobon (2002), I will define contagion as an increase in cross market correlation³ during periods of distress.

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¹Due to sharp rise of market uncertainty the inter-bank market became dysfunctional, threatening even larger liquidity and bank insolvency crisis.

²Economists cannot even agree on a common definition of economic contagion (for a discussion see Forbes and Rigobon, 2001).

³Traditionally correlation of stock market indices or asset prices were analyzed, but in this study I focus on the co-movement of volatilities of major stock market indices.

This definition is very strict, yet it has an important merit, it leads directly to a simple econometric test of the contagion hypothesis – simply through testing the switch in the strength of this correlation.

This type of market contagion has been thoroughly studied across different countries and across different asset classes. Starting with the seminal paper of King and Wadhvani (1990) and followed by others (see for example Longin and Solnik, 1995), economists found evidence supporting the market contagion hypothesis. On the other hand the studies of Forbes and Rigobon (2002) and Longin and Solnik (2001) claimed that after correcting for estimator bias there is no evidence for market contagion. Finally, Corsetti et al. (2005) showed that even after correcting for estimator biases one cannot reach a sound conclusion on the contagion hypothesis.

All the afore mentioned articles focus on the correlation of returns of key market indices or selected assets. Only recently economists started studying the issue of market risk contagion, by looking directly at realized (Diebold and Yimaz, 2009) or both realized and implied volatilities of stock market indices (Cipollini et al. 2013). Those studies provide support for the presence of volatility contagion during periods of financial stress.

The current literature on economic contagion focuses on testing for the existence of a structural break in correlation during the period of financial turmoil. This is somewhat difficult as the periods of turmoil are usually very short and consequently span only a small portion of the observed sample (Dungey and Zhumabekova, 2001). Moreover the exact choice of dates for the financial turmoil “regime” might also lead to inconsistent or inefficient estimates (Rigobon, 2004).

In this study I suggest a novel approach to test for the contagion hypothesis that circumvents these problems. Instead of analyzing periods of turmoil and comparing them to the tranquil periods, I look directly at the correlations of market pricing of the crash and non-crash risk. More precisely I decompose Variance Risk Premia⁴ into premia attributed to crash and non-crash states. I modify the methodology of Bollerslev and Todorov (2011b) and replicate their results for the S&P500 index. I also extend those calculations to FTSE100 and Eurostoxx50 indices. This allows me to compare the co-movement of the premia for the market crash with the co-movement of premia for the rest of the risk. I find that the market crash premia exhibits higher correlation than the residual premia.

The tail-risk premia correlations are elevated, relative to the correlation of the remainder of the premia, even when I account for the time varying correlation. I use the Dynamic Conditional Correlation model of Engle (2002) to calculate time-varying correlation coefficients, as the cross market correlations are renowned to be unstable over time. I find that the correlations of the crash-risk premia are indeed time-varying, yet they remain relatively stable over time

⁴Variance Risk Premium is the premium that markets require for the risk of a change of uncertainty. This premium is calculated as a difference between the statistical measure of market volatility (empirically measured by the realized volatility) and the risk neutral implied volatility (empirically measured by the options implied volatility index, ex. VIX).

(implying high persistence of correlations). This is especially striking in comparison with the fact that the crash-premium itself is very sensitive to key market events (e.g. Russian default, LTCM collapse, Lehman Brothers bankruptcy etc.).

Summing up, in my study I have decomposed Variance Risk Premia for three major equity indices into their market crash and non-crash components. I find that the most of the variance premium is determined by the crash risk on all the studied markets. This result is consistent with vast theoretical finance literature of the large impact of the crash risk on both dynamics and level of the risk premia (see Rietz 1988, Barro 2006 or Gabaix 2012).

Moreover, to the best of my knowledge, this paper is the first one to show that the volatility premia demanded for the crash risk are more tightly co-moving across the markets than the premia for the non-crash risk.⁵ This apparent higher correlation of the premia for the tail events implies that investors view the tail events as the ones that have global impact. This in turn means that financial markets are pricing in market contagion i.e. the fact of higher co-dependence of largely adverse shocks, at least according to the definition of contagion followed in this paper.⁶

The tail-dependency of volatility premia documented in this study has a straightforward implications for investors' portfolio design. It shows that potential gains from portfolio diversification are smaller than what could be expected when not accounting for tail-dependency, as the cross-country hedging will not be effective during the times of turmoil. This may lead to a conclusion that models that do not capture tail dependence of the variance premia will overestimate diversification returns. This, in turn, might bias the estimates of the home bias upwards. Moreover, Bollerslev et al. (2014) showed that the tail risk premium is a good predictor of future equity returns of the S&P500. My study implies that the majority of the tail-premium is determined globally, hence it should also exhibit predictive power over the other equity markets. This implies that global tail risk premium might be an important pricing factor.

Olivier Blanchard, the chief economist of the IMF, said that policy makers should remove tail-risks and perceptions of tail-risks.⁷ This relates this study to the issue of the risk taking channel of the monetary policy. Classically this issue is viewed from the perspective of bank's willingness to lend money for the riskier projects. Yet, it can also be mirrored in the pricing of the risk in the equity markets. Higher propensity of banks to finance riskier projects would imply lower risk premia on equity markets. Consequently one could link monetary

⁵Bollerslev et al. (2012) or Londano (2012) show that the Variance Risk Premia are dominated by a global component, yet they do not look into the split of the VRP into the tail- and non-tail risk related premia.

⁶As mentioned before this is a very restrictive definition of contagion. This definition does not provide any details on the channels of contagion. Moreover it also does not take into consideration the problem of the common shocks. In fact, for this definition to pin down contagion solely one has to assume that common shocks have the same volatility as the idiosyncratic shocks. This is one of the reasons why Forbes (2012) claim that contagion should not be analysed from correlation perspective.

⁷The Economist, January 31, 2009

policy with the risk appetite of the investors for bearing the tail risk. Hattori et al. (2013) studied the impact of the quantitative easing (QE) on tail risk perceptions, finding statistically significant decrease of tail premia due to the QE on the US market. My analysis, however shows that tail premia are co-moving closely across different countries, hence any policy that reduces tail-risk premia should have a global impact. This implies that US QE might have large spillover effects on other equity markets and consequently on other economies. The analysis developed in this study suggest that an interesting direction for future research is to investigate the global aspect of the QE.

The remainder of the paper is organized as follows. Section 2 briefly describes the methodology and Section 3 characterizes the dataset used for the analysis. Section 4 describes the results and the last section concludes.

2 Methodology

The methodology of this study composes of three parts. First, I provide intuition on the concept of the Variance Risk Premium (VRP) and I show how it is measured using daily data on options and 5 minute intra-day data on futures. Second, I describe how to decompose VRP into the part related to the tail risk and the part related to the non-tail risk, using techniques developed by Bollerslev and Todorov (2011b). I also include description of my modification that allows to use the original methodology with other datasets. Finally, I lay down the Dynamic Conditional Correlation model of Engle (2002), which is used to analyse potentially time-varying correlations between different premia.

2.1 Variance Risk Premium (VRP)

Many financial studies have shown that not only equity returns, but also volatilities (risks) of those returns are time-varying. This basic fact of non-constant volatility means that this is an additional source of investment risk. In fact markets are pricing this risk in the form of the Variance Risk Premia (VRPa). This is a relatively new concept describing market's premium for volatility instability. Yet, financial markets have already developed tools to hedge this type of risks. VRPa can be traded using variance swaps (see Demeterfi et al. (1999) or Levin (2014) for details). Those instruments simply swap future unknown realized variance for current option implied variance.

On the technical side, the VRP is measured as the difference between the physical expectations (the P-measure) of the returns' realized quadratic variation and the risk neutral expectations (the Q-measure) of the quadratic variation.

$$VRP_t = \frac{1}{T-t} \left(E_t^P(QV_{[t,T]}) - E_t^Q(QV_{[t,T]}) \right) \quad (1)$$

The physical expectations (the P-measure) of the quadratic variation is simply the best statistical $T-t$ periods ahead forecast. Quadratic variation is

measured as the realized variance based on the 5 minutes intra-day prices of index futures. This approach has been strongly advocated by Shepard et al. (2013), who showed that this is the best variance estimator. In order to adjust for the overnight price changes daily realized variance is rescaled by the constant proportion of overnight change. Moreover in this study, following Bollerslev et al. (2009), I use simple naïve expectations of the realized variance as a proxy for realized variance forecast. This approach should be effective as variance exhibits large persistence, exemplified by the volatility clustering effects.⁸

$$E_t^P(QV_{[t,T]}) = \sum_{i=t-(T-t)}^t RV_i \quad (2)$$

The risk-neutral expectations of the quadratic variation (the Q measure) is measured using daily data on the panel of options. Those data enable us to calculate the model free option implied variance of future prices. This type of variance measure simply reflects the value of the expected variance under the assumption of risk neutrality of market participants. In more technical terms this measure assumes that the stochastic discount factor is constant and equal to the inverse of the risk-free interest rate. This means that, in case of risk-averse investors, the Q measure of the variance combines investors' expectations of future variance with their risk preferences (see Figlewski 2012).⁹ The most classical example of a model free Q-measure of volatility is the VIX index.¹⁰

My Q measure of the quadratic variation only slightly differs from the VIX index.¹¹ Both measures use interpolation/extrapolation to calculate implied volatility for a fixed time horizon. Yet, unlike the VIX which uses only two different option maturities to calculate intrapolated/extrapolated values, I use the whole available set of different maturities of options. Moreover, in contrast to the VIX methodology which interpolates/extrapolates linearly quadratic volatility, I interpolate/extrapolate option prices using Carr and Wu (2003) polynomial and based on theoretical option prices I calculate the implied volatility.¹² This change in the calculation method is motivated by two facts. First, the set of

⁸More recently, however Bekaert and Hoerova (2014) or Kaminska and Roberts-Sklar (2015) show that the naïve forecast can be improved if the forecasting method models separately continuous and jump part of the volatility. Furthermore the forecast might be improved even more by the use of option implied volatility data. Yet, given that the focus of this study is the decomposition and cross correlation of VRPa, it seems that simple naïve expectations forecast would work well.

⁹Simple coin flipping game might be a great example to understand the difference between Q- and P- measure of the probability distributions. Say, the game pays EUR 100 in case the flip yields heads and 0 in the other case. The P -measure would correspond to the actual distribution, hence both events have probabilities equal to 0.5. In order to determine the Q-measure of probabilities we need to know the price of the game. Say, an economic agent is willing to pay EUR30 for that game. Under the assumption of risk-neutrality this would mean that the distribution of the probability should be 0.3 for heads and 0.7 for tails. The difference between those two measures of probabilities simply reflects agents risk aversion.

¹⁰This index just has to be divided by 100 and squared to obtain implied variance.

¹¹In fact the correlation of my measures with volatility indices: VIX, VFTSE and VStoxx is very high and amounts roughly to 95%.

¹²Please refer to the Appendix A for more details on the approximation.

data used in this study, suffers from a small number of very close to maturity options, hence the VIX methodology would imply linear extrapolations from the two options with quite distant maturities. This seems very inappropriate, especially when dealing with options capturing large jump probabilities. Second, I wanted to keep my measure consistent with the decomposition of the VRP presented in the following section.

Equation 3 describes the formula for the Q measure of the quadratic variation, once, the theoretical 14-day to maturity options are calculated:

$$E_t^Q(QV_{[t,T]}) = \frac{2}{T-t} \sum_i \frac{\Delta K_i}{K_i^2} e^{(T-t)r} Q(K_i) - \frac{1}{T-t} \left[\frac{F}{K_0} - 1 \right]^2 \quad (3)$$

In my calculations options time to maturity $T-t$ is fixed to 14 days (it is always quoted as a fraction of a year). The forward index level F is calculated based on the index level at a given moment and the respective (14 day) risk-free interest rate r . K_0 denotes the first strike price below the forward index level F of the panel of options. K_i is the strike price of i th out-of-the-money option; a call if $K_i > K_0$ and a put if $K_i < K_0$; both put and call if $K_i = K_0$. ΔK_i is simply a mid-point between two strike prices: K_{i-1} and K_{i+1} . The price of the option $Q(K_i)$ for a given strike price is either a price of the call option $C(K_i)$ if $K_i > K_0$ or a price of a put option $P(K_i)$ if $K_i < K_0$. The entire equation 3 is exactly the same as the one used to calculate the VIX index (see Chicago Board Options Exchange White Paper).

Finally, as shown in eq 1, VRP is measured as the difference between the two expectations, hence it reflects investors' attitude towards the risk – the so called risk appetite. The decomposition of this risk enables us to understand what drives those premia: tail-events or more “normal” type of equity return movements. In the next section I lay down the basic assumptions on the asset price dynamics needed to calculate how much of the VRP is attributed to the tail events.

2.2 Tail-premia measures

Bollerslev and Todorov (2011b) methodology, which is applied in this paper, requires that the underlying asset prices follow a very general jump-diffusion process.¹³ It implies that the asset price dynamics follows stochastic differential equation:

$$\frac{dF_t}{F_t} = \alpha_t dt + \sigma_t dW_t + \int_R (e^x - 1) \tilde{\mu}(dt, dx) \quad (4)$$

¹³This type of process is very common in the financial literature, mainly due to the fact that it fits the actual data very well. Moreover, it allows prices to exhibit discontinuous patterns, which in turn, justifies the existence of markets for financial options in theoretical finance models (for some discussion of merits of jump-diffusion models please refer to Tankov and Voltchkova(2009)).

where α_t denotes the drift, σ_t denotes the instantaneous volatility and W_t is a standard Brownian motion. The first two elements of the sum depict the continuous part of the dynamics. The third part of the sum describes jumps or discontinuities of the asset price dynamics, where the $\tilde{\mu}(dt, dx)$ is the so-called compensated jump measure. The jump part may for example follow a Poisson process as in the Merton (1976) model. But in case of this study there is no need to limit ourselves to any parametric distribution, neither for the continuous, nor for the jump part. In fact, for our further analysis, the most important feature of this model is the additive separability of the continuous and the jump components.

Both the diffusion and the jump part of the asset price dynamics will have their parallels in the process describing asset price variance. Let us, consider quadratic variation of the logs of asset prices over the $[t, T]$ time interval:

$$QV_{[t, T]} = \int_t^T \sigma_s^2 ds + \int_t^T \int_R x^2 \mu(ds, dx) \quad (5)$$

where the first component $\int_t^T \sigma_s^2 ds$ is the volatility of the continuous process and the second component $\int_t^T \int_R x^2 \mu(ds, dx)$ denotes the volatility generated by the discontinuous part. In principle the first part should be responsible for the volatility generated by the “smaller” (continuous) movements in the asset prices, whereas the second part would depict volatility generated by the “larger” asset price movements (jumps).

Quadratic variation equation eq 5 implies that the VRP, defined by eq 5, will simply be a sum of two differences: the difference between P and Q expectations of the continuous part of the quadratic variation and the difference between P and Q expectations of the jump part of the quadratic variation

$$\begin{aligned} VRP_t = & \frac{1}{T-t} \left(E_t^P \left(\int_t^T \sigma_s^2 ds \right) - E_t^Q \left(\int_t^T \sigma_s^2 ds \right) \right) \\ & \frac{1}{T-t} \left(E_t^P \left(\int_t^T \int_R x^2 \mu(ds, dx) \right) - E_t^Q \left(\int_t^T \int_R x^2 \mu(ds, dx) \right) \right) \end{aligned} \quad (6)$$

Given that in this study the main focus is on the tail-risk, we need to create a variance premium measure that would only capture the premium for tail events. Bollerslev and Todorov (2011b) propose to define a $VRP(k)$ that only captures volatility premia induced by asset price moves above substantially large threshold k :

$$\begin{aligned} VRP_t(k) = & \frac{1}{T-t} \left(E_t^P \left(\int_t^T \int_{|x|>k} x^2 v_s^P(dx) ds \right) \right) \\ & - \frac{1}{T-t} \left(E_t^Q \left(\int_t^T \int_{|x|>k} x^2 v_s^Q(dx) ds \right) \right) \end{aligned} \quad (7)$$

where the k parameter denotes a threshold separating large jumps from the small ones. It might be quickly noted that the $VRP_t(k)$ only depends on the jump part of the process. This approximation is of course valid only for sufficiently high k .

Finally on the basis of the $VRP(k)$ and the total VRP , I can also define a truncated volatility measure $VRP(Tr)$. This measure will capture the size of the variance risk premium that is not attributed to the market crash:

$$VRP_t(Tr) = VRP_t - VRP_t(0.9) \quad (8)$$

Moreover, I can also define a measure of VRP that captures all the volatility risk except for the risk of large jumps, both down and upside:

$$VRP_t(Tr - 2) = VRP_t - VRP_t(0.9) - VRP_t(1.1) \quad (9)$$

Having defined tail-risk premia, the next two sub-sections will briefly describe how to calculate Q- and P- measures from the data.

2.2.1 Risk-Neutral (Q) Measures

The most difficult part of the Q-measure estimation is to pin down the process of the jump density $v_t^Q(dx)$. This jump density should be estimated non-parametrically from the options data. The key idea is to generate a time-varying measure with as few assumptions regarding its structure as possible.

$$v_t^Q(dx) = (\varphi_t^+ 1_{\{x>0\}} + \varphi_t^- 1_{\{x<0\}})v^Q(x)dx \quad (10)$$

where φ_t denotes an unspecified stochastic process of temporal variation of the jump arrivals and $v^Q(x)$ is an unspecified time-invariant density. Yet, the methodology of Bollerslev and Todorov (2011b) allows us to estimate tail-volatilities $E_t^Q \left(\int_t^T \int_{|x|>k} x^2 v_s^Q(dx) ds \right)$ even under those very general assumptions. First of all they calculate model-free risk neutral measures from the panel of options data. Second, using the Extreme Value Theory (EVT) those measures are used to estimate Generalized Pareto Distribution (GPD) parameters (namely: scale (σ) and shape (ξ) parameters) and the average jump intensities $E(\frac{1}{T-t} E_t^Q(\int_t^T \varphi_s ds))$ through a just identified GMM estimation. This allows us to fully describe the time invariant part of the jump intensity $v^Q(x)$. Third, using fixed parameters for the GPD, the time varying jump intensities are backed out to fulfill exactly the moment conditions. Finally, using the estimated parameters the Q measure of the tail-volatility is calculated for a given threshold k .

Risk neutral jump-tail measures I describe the risk neutral jump-tail measures in detail as here I deviate slightly from the original Bollerslev and Todorov (2011b) framework.

Bollerslev and Todorov (2011b) propose model-free risk-neutral jump tail measures:

$$RT_t^Q(k) = \frac{e^r C_t(K)}{(T-t)F_t} \quad (11)$$

$$LT_t^Q(k) = \frac{e^r P_t(K)}{(T-t)F_t} \quad (12)$$

where $k = \ln(\frac{K}{F})$ is the log-moneyness, $C_t(K)$ and $P_t(K)$ are prices of call and put options respectively, K is the option strike price and F_t is the price of the underlying futures. Those measures capture solely the jump risk as long as two conditions are fulfilled. First the options have to be deeply out of the money. Bollerslev and Todorov (2011b) use moneyness levels of $\{0.9000\ 0.9125\ 0.9250\}$ for the left tail and $\{1.0750\ 1.0875\ 1.1000\}$ for the right tail, which should guarantee enough distance from the underlying to capture only the jump risk. Second the option needs to be close to maturity. Bollerslev and Todorov (2011b) use options that have median of 14 days to maturity. In my calculations I follow the same levels of option moneyness, but the dataset used in this study has much longer median maturity of options (see Table 7). This means that my model-free risk-neutral jump tail measures might be “contaminated” by the diffusive part of the process. In fact, my jump tail measures were substantially larger when the options had longer maturities relative to the original study.

In order to circumvent this problem I use options with different maturities for a given moneyness to fit the polynomial describing time-decay plot of option price. Carr and Wu (2003) show that a simple polynomial should allow to approximate time-decay of options no matter whether the underlying process contains jumps or not. This approximation allows me to calculate the theoretical price of the 14-days-to-maturity option. Appendix A provides details on the approximation method as well as some robustness checks.

Once I have the theoretical 14-days-to-maturity option price, I construct the same risk-neutral jump tail measures. In this case the pattern of my jump tail measures closely resembles the original one of Bollerslev and Todorov (2011b).

Generalized Pareto Distribution (GPD) parameters estimation and tail-volatility GPD parameters are estimated using the simple non-linear GMM procedure of Hansen and Singleton (1983). The exact moment conditions are described in the Appendix B. The basic principle is that for both tails (left and right) I have 3 parameters to estimate and jump-tail measures for 3 different levels of moneyness, hence the system is just identified.

2.2.2 Objective (P) Measures

Analogously to the Q measure estimation, the key in estimation of the P measure is to pin down the jump density v_t^P . Unfortunately it is not possible to estimate the intensity fully non-parametrically, simply because I do not have three different points of the curve on the same day. Consequently I need to assume some sort of affine model of the intensity. Following Bollerslev and Todorov (2011a) I assume that the temporal variation of the volatility is a function of the stochastic volatility σ_t^2 of the continuous part:

$$v_t^P(dx) = (\alpha_0^- 1_{\{x < 0\}} + \alpha_0^+ 1_{\{x > 0\}}) + (\alpha_1^- 1_{\{x < 0\}} + \alpha_1^+ 1_{\{x > 0\}}) \sigma_t^2 v^P(x) dx \quad (13)$$

This directly implies that I have to estimate four constant across time parameters (namely: scale (σ) and shape (ξ) parameters of the GPD that characterizes $v^P(x)$, and α_0 and α_1) for each tail. Moreover I have to get the estimate

of the time-varying stochastic volatility σ_t^2 . In order to do so I follow directly Bollerslev and Todorov (2011a) .

First I estimate continuous volatility using Mancini’s (2001) idea of truncated volatility. All intra-day asset price movements below a certain threshold contribute to the continuous volatility whereas the ones above the threshold contribute to the jump volatility. The truncation threshold is time-varying to capture the effects of the volatility clustering. The threshold is a function of the past continuous volatility. Moreover the daily pattern of the volatility is also taken into account. For each index I estimate the average volatility for a given exact time. On that basis I calculate the time of the day volatility multiplier that either increases or decreases the threshold. For more details on the realized volatility calculations please refer to the Appendix C.

Second I select a threshold level, which is always higher than the maximum threshold used to determine continuous volatility. I select a threshold of 0.6% for all the indices. On the basis of this threshold I can mark observations that are definitely jumps in the whole sample. Then I use estimated continuous volatility along with matrices indicating jumps (the ones determined by 0.6% threshold) to estimate all four parameters in question. Again the estimation is done using the GMM framework (for details on the exact moments specification please refer to the Appendix D).

Finally, once all the parameters are calculated I calculate the tail-volatilities $E_t^P \left(\int_t^T \int_{|x|>k} x^2 v_s^P(dx) ds \right)$ for the thresholds of 0.9 and 1.1 to match the data for the Q-measure. Again I can compare the GPD parameters across different indices.

2.3 Co-movement measures

In the last step of the analysis I look at the co-movement of the selected indicators (namely: VRP(0.9) and VRP(Tr)). In order to keep the analysis simple and yet powerful all the measures are based on the simple r-Pearson correlation coefficient. Unconditional correlations of selected indicators across three indices (S&P500, FTSE100 and Eurostoxx50) constitute a natural benchmark for further analysis. Moreover, I also report Kendall’s rank correlation coefficients as the relationships between VRPs might exhibit non-linear patterns.

Finally, in order to allow for a more complex dynamic correlation structure, I apply the Dynamic Conditional Correlation (DCC) model of Engle (2002). This model helps me not only to overcome the problem of time-varying correlation, but also helps me to control for the heterogeneity of individual shocks. The model looks at the conditional correlations of innovations, consequently enabling me to gauge how does a shock propagate across the markets.

The DCC model requires certain doze of parsimony, hence the level equations are modeled as a simple AR(4) processes (i.e. the A_i matrices in eq. 14 are diagonal). The choice of this lag is motivated by the potential existence of monthly effects in the premia.

Conditional covariance matrix (eq. 15) is decomposed into the matrix of

individual conditional standard deviations D_t and conditional correlation matrix R_t (see eq. 16). Conditional standard deviation matrix D_t is a diagonal matrix where each element on the diagonal simply represents a square root of individual variances which are modeled as the GARCH(1,1) process. Transformation of the conditional correlation matrix (see eq. 17) guarantees that the matrix has ones on the diagonal. Quasi conditional correlation (see eq. 18) is a weighted average of the unconditional sample correlation \bar{R} (see eq. 19) and the previous period cross product of 'corrected' innovations (see eq. 20) and the previous period conditional quasi correlation. The specification of the eq. 18 nests the Constant Conditional Correlation (CCC) model of Bollerslev (1990), hence allowing for direct testing of the time varying correlation assumption. Should λ_1 and λ_2 parameters were jointly statistically insignificant, then the correlation between innovations would be constant over time.

$$y_t = C + \sum_{i=1}^4 A_i y_{t-i} + \epsilon_t \quad (14)$$

$$E_{t-1}(\epsilon_t \epsilon_t') = \Sigma_t \quad (15)$$

$$\Sigma_t = D_t R_t D_t \quad (16)$$

$$R_t = \text{diag}(Q_t)^{-1/2} Q_t \text{diag}(Q_t)^{-1/2} \quad (17)$$

$$Q_t = (1 - \lambda_1 - \lambda_2) \bar{R} + \lambda_1 \tilde{\epsilon}_{t-1} \tilde{\epsilon}_{t-1}' + \lambda_2 Q_{t-1} \quad (18)$$

$$\bar{R} = E[\tilde{\epsilon}_t \tilde{\epsilon}_t'] \quad (19)$$

$$\tilde{\epsilon}_t = D_t^{-1} \epsilon_t \quad (20)$$

3 Data

The dataset used in this study allows me to replicate the US results of Bollerslev and Todorov (2011b) as well as to extend their calculations to the UK and the Euro-zone. Accordingly, US calculations are based on the S&P500 index, the UK on the FTSE100 index and the Eurozone on the Eurostoxx50 index. The Q measure (implied distribution) is based on a daily panel of options, whereas the P measure (statistical measure) is based on intra-day (5 minutes) data on traded futures, obtained from Thomson Reuters. Finally, correlation calculations are conducted on the weekly averages, as the daily data contained too much noise.

3.1 Options

I use options data provided by the Bank of England. The data are sampled with a daily frequency. The data for S&P500 and FTSE100 options span from January 1995 to December 2012. Unfortunately the data span for the Eurostoxx50 is shorter and covers only dates from January 1999 to December 2012. This sample still allows me to cover major market turmoils (LTCM, Russian and Asian crises - for US and UK only; dotcom bubble burst, accounting scams and the great recession period for all indices).

I apply a standard set of filters on the options data before any calculations take place. The set of filters is based on programmes used by the Bank of England which are in line with the ones used in Carr and Wu (2009). The detailed set of filters used with their brief description can be found in the Appendix E.

3.2 Intra-day data

I use the intra-day data provided by Thomson Reuters. The data are sampled with 5-minutes frequency. This frequency allows me to capture price jumps as well as it should allow to limit the impact of the microstructural noise. In fact Shepard et al. (2013) show that realized variance based on 5-minute data is the best estimator of the realized variance across different assets.

For S&P500 and FTSE100 I use the data spanning from January 1996 to December 2012, whereas for Eurostoxx50 the data only spans from January 1999 to December 2012. The range of the dataset for the S&P500 is unfortunately shorter than in the Bollerslev and Todorov (2011b) paper, hence the parameter estimates might differ. In terms of trading time I have tried to pick hour brackets between which I had data throughout all the dates. Consequently, my time window is: for S&P500 - 81 observations (from 8.30 to 15.10), for FTSE100 - 94 observations (from 8.15 to 16.00) and for Eurostoxx50 - 81 observations (from 9.15 to 15.55).

4 Results

4.1 Q-measure

Table 1 summarizes parameter estimates for the risk-neutral Q measure. It is clear that the implied distributions are skewed. Left tails are much heavier across all the indices. This is simply a confirmation of the existence of the so called volatility “smile”.

The left-tail seems to be the lightest for the FTSE100 index, as all the parameters are the smallest across all three indices. Eurostoxx50 exhibits the heaviest tail, but the curvature of the tail is slightly smaller than the one of the S&P500. This means that the distance between Eurostoxx50 and S&P500 will diminish further in the tail. The results for the right tails also seem to follow the same pattern. It is more informative to look at the annualized average jump intensities presented in Table 2. Those intensities again underline the skew of

	S&P 500		Eurostoxx 50		FTSE 100	
	LT	RT	LT	RT	LT	RT
ξ	0.2570	0.0615	0.2247	0.0815	0.2107	0.0278
σ	0.0513	0.0242	0.0618	0.0370	0.0500	0.0263
αv	1.1431	0.7266	1.4813	0.2457	0.9761	0.6465

Table 1: Q measures estimation results.

Jump Size	US	Euro	UK
>7.5%	0.7266	1.2457	0.6456
>10%	0.2666	0.6457	0.2526
>20%	0.0082	0.0632	0.0074
<-7.5%	1.1431	1.4813	0.9761
<-10%	0.6627	0.9338	0.5542
<-20%	0.1052	0.1758	0.0760

Table 2: Q measure - annualized jump intensity estimates.

the distribution. The difference between the left and right tail jump intensities is even more pronounced for the more extreme jumps. Finally, the magnitudes of the average annualized jump intensities seem to be quite high and not really matching the actually observed data.

4.2 P-measure

Table 3 summarizes estimation results for the objective P measure. Unfortunately the estimates of the GPD parameters are not comparable with the ones from the Q measure, as they were taken at a different threshold. Yet, I can briefly characterize the behavior of tails under the P measure.

First of all a small skew towards the left tail can be noted. This stands in contrast with the Bollerslev and Todorov (2011a) findings, who note a skew towards the right tail. It can be explained by the fact that the sample I use also covers the period of the great recession. Second, the tail ordering in terms of tail heaviness seems to be reversed relative to the Q measure. Now the Eurostoxx50 seems to have the lightest tails and FTSE100 seems to have the most leptokurtic distribution. This is particularly noticeable when analyzing the values of the average annualized jump intensities (see Table 4).

It should also be noted that all the jump intensities seem to be closely connected with the continuous volatility, as all the estimates of α_1 remain positive and different from zero.

A quick comparison of the average annualized intensities for the P measure (see Table 4) with those for the Q measure (see Table 2) clearly suggests that the actual jump probabilities are much smaller. This happens both for the right and left tails of the distribution.

	US		Euro		UK	
	LT	RT	LT	RT	LT	RT
ξ	0.2500	0.2088	0.2305	0.1648	0.2596	0.2218
100σ	0.1594	0.1834	0.1819	0.1955	0.1624	0.1714
α_0	-0.0016	-0.0019	-0.0016	-0.0013	-0.0025	-0.2772
α_1	0.0329	0.0396	0.0346	0.0290	0.0406	0.0402

Table 3: P measure estimation results

Jump Size	US	Euro	UK
>7.5%	0.0062	0.0016	0.0187
>10%	0.0016	0.0003	0.0052
>20%	0.0001	0.0000	0.0002
<-7.5%	0.0069	0.0082	0.0343
<-10%	0.0020	0.0022	0.0102
<-20%	0.0001	0.0001	0.0004

Table 4: P-measure jump intensities

4.3 Variance Risk Premia, Tail Risk Premia and Investor Fears Indices

On average all the Variance Risk Premia (VRP) are negative. They also exhibit substantial volatility (see table 5). Moreover at the periods of turmoil they all seem to go through two phases: first when a sudden increase of the realized volatility is not matched by the implied volatility (hence an increase in the VRP) and second a sharp increase in the implied volatility with a normalization of the realized volatility leading to a sharp decrease in the volatility (see Figure 1).

Figure 2 shows the dynamics of the estimated $VRP(k)$. The values of the $VRP(0.9)$ related to the negative jump of 10% are clearly higher in the absolute values than the values of $VRP(1.1)$ of the 10% positive jump - once again depicting the distribution's skew towards negative values. Moreover it seems that the Eurostoxx50 reacts to the unfavorable news much more than the other markets. The absolute value of the $VRP(k)$ for the Eurostoxx50 in periods of turmoil is much higher than in the other two indices.

$VRP(k)$ dynamics seems to match significant market events such as LTCM bankruptcy, Russian default, dotcom bubble burst and economic downturns.

	Mean	Standard Deviation
S&P500	-0.0232	0.0222
FTSE100	-0.0081	0.0402
Eurostoxx50	-0.0244	0.0310

Table 5: Summary statistics for Variance Risk Premia, based on weekly data

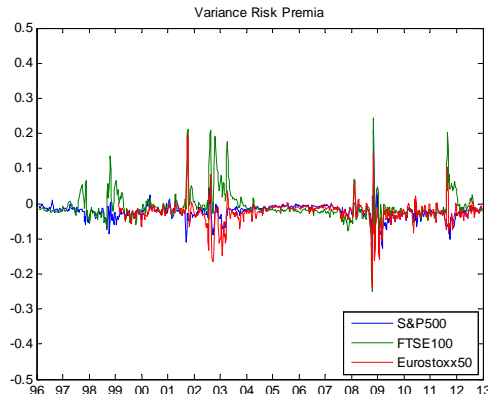


Figure 1: Variance Risk Premia, based on the weekly data

Consequently the absolute value of the $VRP(k)$ increases during the Russian crises, at the dotcom bubble burst, and during the Great Recession. All indices noted the biggest dip in $VRP(k)$ during the Great recession, but in the case of S&P500 the jump was much higher than any noted before. Another interesting fact is the prolonged increase in the $VRP(k)$ for the Eurozone in 2003. This may be caused by the fact that the Eurozone at that period was suffering from an economic slowdown. Finally the last interesting aspect pertains to the period of the Eurozone sovereign debt crisis. It is clear that the Eurostoxx50's $VRP(k)$ reacted by much more than the S&P500's $VRP(k)$, where this period was hardly noticeable.

It is easy to note that both downside and upside tail $VRP(k)$ measures react at the same time. Consequently one might relate it solely to an increase of overall jumps intensities in the economy. That is why it is also useful to look at the downside premium “corrected” by the premium coming from the upside risk. Following Bollerslev and Todorov(2011b) I define an Investor Fears Index as:

$$FI_t(k) = VRP_t^-(k) - VRP_t^+(k) \quad (21)$$

Investor Fears Indices co-move clearly across different markets. Moreover they are very sensitive to major market events, reaching very low values during market turbulences (see Figure 3). In addition, periods when those indices drop to the lowest values coincide with the ones when $VRP(0.9)$ reaches their lowest values.

4.4 New evidence on contagion

From the perspective of the volatility contagion the most important is to compare different indices correlations of $VRP(0.9)$ to correlations of $VRP(Tr)$. Ta-

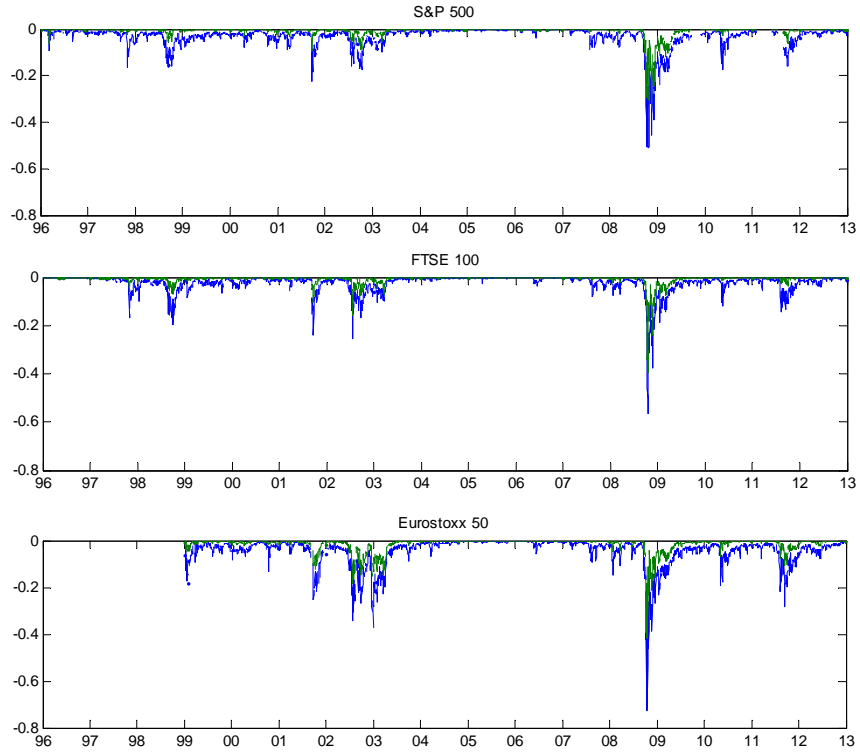


Figure 2: VRP for $k=0.9$ and $k=1.1$

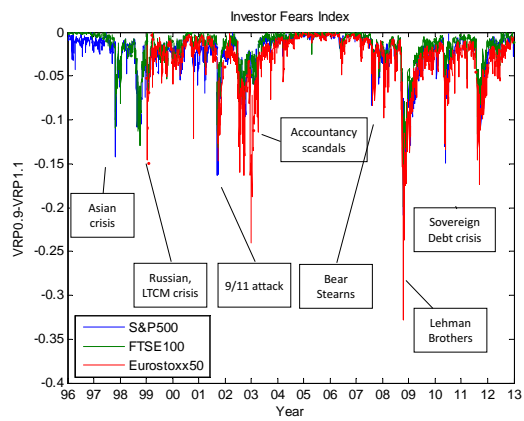


Figure 3: Investor Fears Index ($VRP(0.9) - VRP(1.1)$) vs. major adverse events.

	r-Pearson		Kendall τ	
	VRP(Tr)	VRP(0.9)	VRP(Tr)	VRP(0.9)
S&P500 FTSE100	0.7089	0.9476	0.3290	0.7528
S&P500 Eurostoxx50	0.7930	0.8977	0.4409	0.7418
FTSE100 Eurostoxx50	0.8989	0.9238	0.5464	0.7586

Table 6: Pairwise correlations of the VRP(0.9) and VRP(Tr), based on the weekly data.

ble 6 show that there is a notable increase in correlations for volatility tail premium VRP(0.9) relative to the truncated volatility premium VRP(Tr). This phenomenon shows that markets price in market contagion. Moreover it also underlines that the tail premium is more globally determined relative to the rest of the premium.

VRPs seem to be quite volatile and heavy-tail distributed, hence for robustness I also look at Kendall's τ measures. This measure is more immune to outliers than r-Pearson correlation coefficient. Yet, calculated Kendall's τ 's exhibit even more striking evidence for market contagion hypothesis, as the gap between measures for VRP(Tr) and VRP(0.9) is even wider.

Market correlations are perceived to be unstable over longer periods of time. In order to overcome this problem I have extended my analysis to the dynamic correlations. A simple rolling window analysis, presented in Appendix F, shows that correlations in question are indeed unstable. Moreover, Forbes and Rigobon (2002) pointed out that this type of simple analysis might be biased due to heterogeneity of individual shocks.

As mentioned earlier, I solve both aspects by looking at the AR(4)-DCC(1,1) model. This class of models shows that the conditional correlation of dynamic innovations is time-varying, yet the correlations for the tail premia (VRP(0.9)) remain higher than the ones for the reminder of the premia (VRP(Tr)) (see Figure 4). The gap between the innovation correlations is the biggest for S&P500 - FTSE100 pair. For this pair also the VRP(0.9) innovation correlation exhibits the highest volatility. For the two other pairs the relationships remains much more stable. Still the coefficients of the dynamic correlation part remain significant, rejecting the Constant Conditional Correlation model of Bollerslev et al. (1988). For more details about each pair's estimates please look at the Appendix G.

One of the key drawbacks of Dynamic Conditional Correlation models is the fact that they are very dependent on the specification of the level equation. Different specifications may lead to different results or even to unfeasible solutions (mainly due to non-finding semi-positive definite variance-covariance matrices). I have also noted this problem in my analysis, while trying different level specifications (simple mean model, AR(1), AR model with lagged values of the other indices). Still the key feature of the data remained unchanged, i.e. tail risk premia (VRP(0.9)) are more correlated than the truncated risk premium (VRP Tr), even though the levels of shock correlations differed.

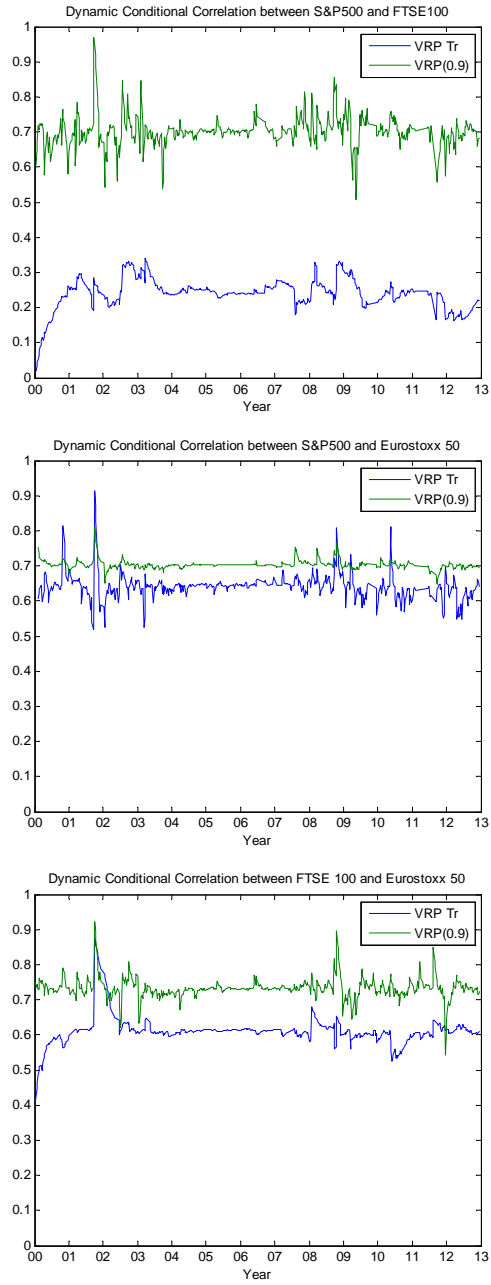


Figure 4: Time varying conditional correlations calculated on the basis of weekly data by AR(4)-DCC(1,1) model.

Index	min	max	median
BT - S&P 500	5	x	14
S&P 500	6	75	39
FTSE 100	5	29	15
Eurostoxx 50	5	74	36

Table 7: Maturities of the closest to maturity options. Source: Bollerslev and Todorov, 2011 and own calculations.

Finally one may also challenge the tail measure (VRP(0.9)) as it strongly co-moves with the upside tail measure (VRP(1.1)). That is why I have also compared correlations and dynamic correlations of Investors Fear Indices with VRP(Tr-2) measures (i.e. VRP-VRP(0.9)-VRP(1.1)). The qualitative results remained unchanged (see Appendix H).

5 Conclusions

I managed to calculate VRP and VRP tail measures (VRP(0.9) and VRP(1.1)) for three major equity indices: S&P500, FTSE100 and Eurostoxx50. I found evidence showing that financial markets price in market volatility contagion. This result remains in place even when I allow for a dynamic correlation and when I control for individual variance heterogeneities. Moreover, I showed that VRP tail measures correlation is quite stable (though dynamic) and does not sharply react to major market events. This contrast to the VRP tail measures, which react strongly to key market events.

Appendices

Appendix A - Time-decay approximation

The dataset used in this study has one substantial drawback - the time to maturity of options (that are closest to maturity) is much longer than in the Bollerslev and Todorov (2011b) study (see Table 7), except for FTSE100. Consequently the estimator of the tail measure could be “contaminated” by the diffusion process. This in turn may bias my estimates of the Generalized Pareto Distribution leading to an inaccurate inference about tail-risk premia. In order to circumvent this problem I use all available maturities of options to estimate the time-decay patterns. This allows me to calculate the theoretical value of option that has 14 days to maturity.

Out-of-the-money options at the maturity have zero value. However, the order of convergence over time to that value depends largely on the process governing the underlying asset’s price dynamics. Carr and Wu (2003) showed that the time decay (or the order of convergence) of out-of-the money options is dominated by the presence of jumps. They showed that if the price of the

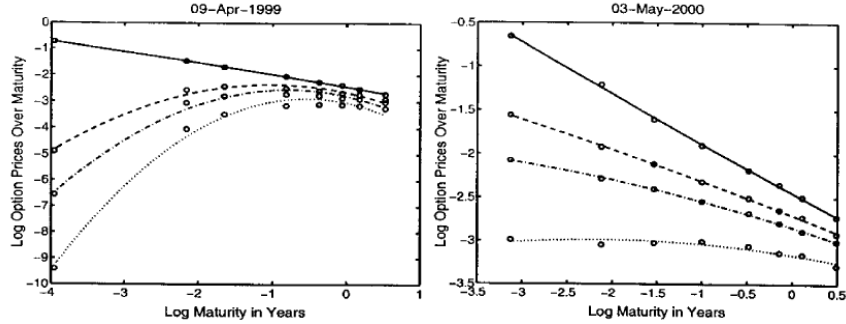


Figure 5: Time-decay plots with fitted polynomial. Source: Carr and Wu (2003)

Number of options	S&P 500	FTSE 100	Eurostoxx 50
6	15%	91%	59%
5	12%	9%	5%
4	52%	0%	5%
3	20%	0%	31%

Table 8: The proportion of maturity nodes in the data

underlying asset follows a jump process or a jump-diffusion prices then the value of the out-of-the-money option will converge more slowly to zero than in the case of a strict diffusion process. They also showed that the time decay of option prices can be closely approximated by the following polynomial:

$$\ln\left(\frac{P}{T}\right) = a(\ln T)^2 + b(\ln T) + c$$

This approximation equation is valid regardless of the underlying process exhibiting jumps or not. If the underlying equity process has no jumps the fitted line should have a greater slope close to the zero maturity, whereas in case it exhibits jumps the time-decay plot should be flatter (see Figure 5). In this study I fit this polynomial for each day of the data since the perception of the jump probability might change over time. The fitted line allows me to calculate the theoretical option value for the exact 14 days to maturity.

The number of options used in the approximation varies over time and is driven by the data availability. I use 3 to 6 option maturities to fit the polynomial - Table 8 shows details for each index. I should expect to get the best results for the FTSE100 index as its option data displays the highest quality - shortest maturities and most of the dataset is covered by 6 maturities. However given that the S&P500 index is the only one present in the original Bollerslav and Todorov (2011b) study I will start my robustness check with it.

First of all it might be noted that the dynamics of tail measures calculated on the bias of the approximation follows nearly the same pattern as the one of

	BT: S&P 500		S&P 500	
	LT	RT	LT	RT
ξ	0.2581 (0.0282)	0.0793 (0.0147)	0.2570 (0.0130)	0.0615 (0.0161)
σ	0.0497 (0.0021)	0.0238 (0.0010)	0.0513 (0.0009)	0.0242 (0.0006)
αv	0.9888 (0.0525)	0.5551 (0.0443)	1.1431 (0.0142)	0.7266 (0.0156)

Table 9: GMM estimates of the Q-tail parameters

Jump Size	US-BT	US
>7.5%	0.5551	0.7266
>10%	0.2026	0.2666
>20%	0.0069	0.0082
<-7.5%	0.9888	1.1431
<-10%	0.5640	0.6627
<-20%	0.0862	0.1052

Table 10: Annualized jump intensities implied by the Q-tail distributions

Bollerslev and Todorov (2011b) (see Figure 6). The two biggest differences are a jump in the tail measure in the early 1996 that is only present in my calculation and a more pronounced response of my tail measure to the dotcom bubble burst in the late 2001. Unfortunately I do not have the original time-series data of the tail measures to compare the accuracy of the fit using some sort of metric. This is why I compare the GMM estimation results (see Table 9). The estimates of the GPD are very close to each other especially for the left tail, as this tail is being estimated with a higher accuracy. The only substantial difference are slightly higher estimates of the jump intensity parameters. However as one may note from the final results of the structure of the jump probabilities the differences are not very large (see Table 10). Judging by the sole comparison of my results to the ones of Bollerslev and Todorov (2011b), it appears that the approximation does a very good job.

Yet, it is still important to see how well the approximation does with other indices. Here I cannot rely on others results, as to the best of my knowledge I am the first one to estimate those measures for other indices. Consequently I have looked at two fit measures and volatility of the theoretical prices for different sets of maturity structures (see Table 11 and Figure 7). The simple fit measure (R^2) does not seem to be a good metric, due to the small number of nodes it will be biased towards high values. The MAPE of the fit evaluated only at the 14 days to maturity also seems to be very small, except for the FTSE100. In that case the MAPE value is ballooned by having a denominator very close to zero. It is very difficult to drive any conclusions from those simple fit metrics as they are based on an insufficient number of data points for each polynomial.

In order to overcome the problem of insufficient number of data points I have looked at theoretical 14-day price volatilities. In principle the volatility of the theoretical price should not depend too much on the set of nodes I use in the

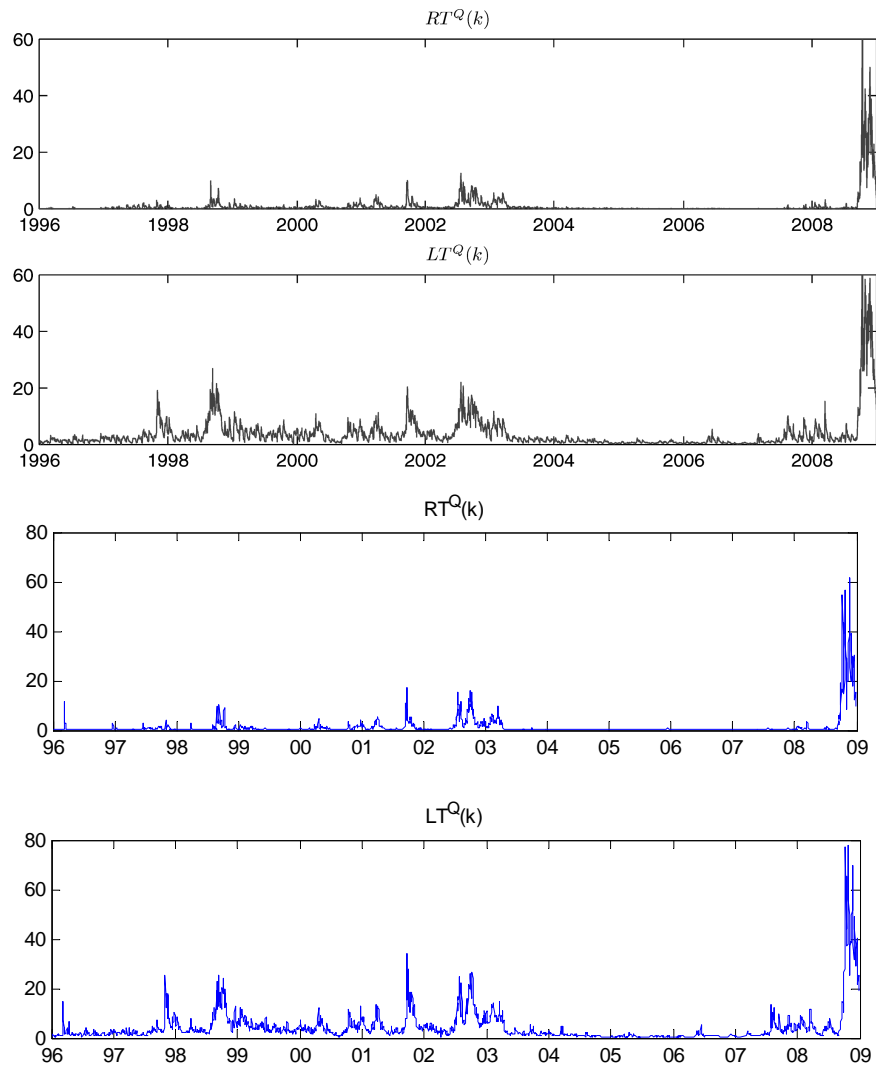


Figure 6: Tail measures for $k=0.9$ and $k=1.1$ for S&P 500. First two graphs are from Bollerslev and Todorov (2011) second from own calculations.

	S&P 500	FTSE 100	Eurostoxx 50
R^2 of the polynomial for 6 different moneyness levels			
Minimum	98.95%	86.04%	99.53%
Average	99.99%	99.91%	99.99%
Percentage error of predicted price for 14-days to maturity option			
MAPE	0.25%	2.86%	0.36%
Maximum	3.47%	56.43%	3.81%

Table 11: The fit of the time-decay polynomial.

approximation. Of course if I extrapolate the 14-day price from a big “distance” the error of fit might generate a higher error than if I have actual maturities very close to the 14 days. Nonetheless it seems informative to investigate how much of the extra volatility is being caused by having distant maturities while performing the approximation. Figure 7 presents inter-quartile ranges for theoretical 14-days prices.¹⁴ The volatility of the theoretical price rises across minimum volatility pointing to certain losses caused by the approximation.

Appendix B - GMM conditions to estimate GPD parameters in the Q measure

The aim of the GMM estimation for the Q measure is to find the following vector of parameters for each tail:

$$\theta^Q = [\alpha_Q^\pm \bar{v}_\psi^{Q^\pm}(tr^\pm); \xi_Q^\pm; \sigma_Q^\pm]$$

fulfilling the following three moment conditions:

$$E(RT_t^Q(k)) = \alpha_Q^+ \bar{v}_\psi^{Q^+}(tr^+) \frac{\sigma_Q^+}{1 - \xi_Q^+} \left(1 + \frac{\xi_Q^+}{\sigma_Q^+} (e^k - 1 - tr^+) \right)^{1-1/\xi_Q^+}$$

$$E(LT_t^Q(k)) = \alpha_Q^- \bar{v}_\psi^{Q^-}(tr^-) \frac{\xi_Q^-}{\xi_Q^- + 1} (e^k)^{1+1/\xi_Q^-} \left(\frac{\xi_Q^-}{\sigma_Q^-} \right)^{-1/\xi_Q^-} *$$

$$*_2F_1 \left(1 + \frac{1}{\xi_Q^-}; \frac{1}{\xi_Q^-}; 2 + \frac{1}{\xi_Q^-}; \frac{tr^- \frac{\xi_Q^-}{\sigma_Q^-} - 1}{e^{-k} \frac{\xi_Q^-}{\sigma_Q^-}} \right)$$

where ${}_2F_1$ is a hypergeometric function and $E(RT_t^Q(k))$ and $E(LT_t^Q(k))$ are sample averages of the introduced tail-measures.

¹⁴Inter-quartile range is being used instead of standard deviations to make the measure more robust to odd observations.

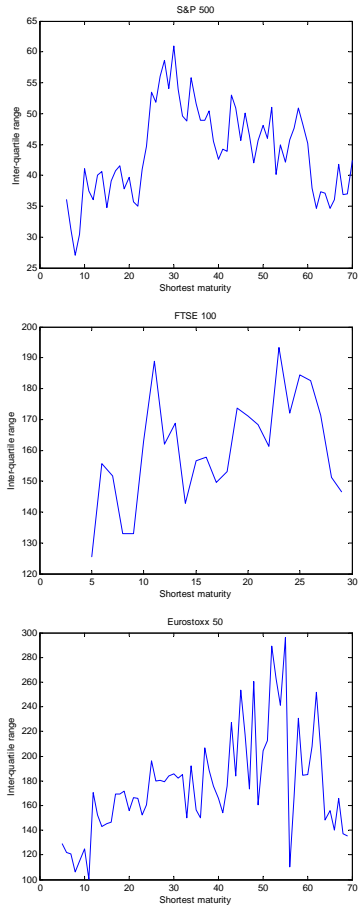


Figure 7: Inter quartile ranges of the theoretical 14-days option prices for different maturities used in approximation. Calculations are based on put out-of-the-money option with $k=0.9$.

Appendix C - Calculation of the truncated volatility

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Appendix D - GMM conditions to estimate GPD and intensity parameters in the P measure

The aim of the GMM estimation of the P measure is to find the following vector of parameters for each tail:

$$\theta^P = [\alpha_0^\pm \bar{v}_\psi^\pm(tr^\pm); \alpha_1^\pm \bar{v}_\psi^\pm(tr^\pm); \xi^\pm; \sigma^\pm]$$

The four moments conditions are as follows:

$$\frac{1}{N} \sum_{t=1}^N \sum_{j=1}^{n-1} \phi_i^\pm(\psi^\pm(\Delta_j^{n,t} p) - tr^\pm) 1_{\{\psi^\pm(\Delta_j^{n,t} p) > tr^\pm\}} = 0 \quad i = 1, 2$$

$$\frac{1}{N} \sum_{t=1}^N \sum_{j=1}^{n-1} 1_{\{\psi^\pm(\Delta_j^{n,t} p) > tr^\pm\}} - \alpha_0^\pm \bar{v}_\psi^\pm(tr^\pm) - \alpha_1^\pm \bar{v}_\psi^\pm(tr^\pm) CV_t = 0$$

$$\frac{1}{N} \sum_{t=2}^N \left(\sum_{j=1}^{n-1} 1_{\{\psi^\pm(\Delta_j^{n,t} p) > tr^\pm\}} - \alpha_0^\pm \bar{v}_\psi^\pm(tr^\pm) - \alpha_1^\pm \bar{v}_\psi^\pm(tr^\pm) CV_t \right) CV_{t-1} = 0$$

where:

$$\phi_1^\pm(u) = -\frac{1}{\sigma^\pm} + \frac{\xi^\pm u}{(\sigma^\pm)^2} \left(1 + \frac{1}{\xi^\pm}\right) \left(1 + \frac{\xi^\pm u}{\sigma^\pm}\right)^{-1}$$

$$\phi_2^\pm(u) = \frac{1}{(\xi^\pm)^2} \ln \left(1 + \frac{\xi^\pm u}{\sigma^\pm}\right) - \frac{u}{\sigma^\pm} \left(1 + \frac{1}{\xi^\pm}\right) \left(1 + \frac{\xi^\pm u}{\sigma^\pm}\right)^{-1}$$

Appendix E - A short guide on how to get ERP(k) and VRP(k) from the GMM estimates

This is a very short and basic instruction on how to derive ERP(k) and VRP(k) for any given threshold k based on estimates. All of the following results are based on the derivations presented in the appendix of the Bollerslev and Todorov (2011b) paper.

Let us have a look at the tail volatility measure first. The measure can be presented as a sum of two components:

$$\int_{x>k} x^2 v(x) dx = 2\bar{v}_\psi^+(tr^+) * K_1 + k^2 \bar{v}_\psi^+(e^k - 1)$$

The first part of the sum is directly determined by my estimates. For the selected threshold of $tr^+ = 0.075$ I have estimated the value directly:

$$\bar{v}_\psi^+(tr^+) = \alpha_Q^+ \bar{v}_\psi^{Q^+}(0.075)$$

The multiplier K_1 is also directly defined by the estimated parameters:

$$K_1 = e^{-k/\xi^+} \xi^+ \left(\frac{\xi^+}{\sigma^+} \right)^{-1/\xi^+} \left[\xi^+ {}_3F_2 \left(\frac{1}{\xi^+}, \frac{1}{\xi^+}, \frac{1}{\xi^+}; 1 + \frac{1}{\xi^+}, 1 + \frac{1}{\xi^+}; \frac{\xi^+ (tr^+ + 1) - 1}{e^k \frac{\xi^+}{\sigma^+}} \right) + k {}_2F_1 \left(\frac{1}{\xi^+}, \frac{1}{\xi^+}; 1 + \frac{1}{\xi^+}; \frac{\xi^+ (tr^+ + 1) - 1}{e^k \frac{\xi^+}{\sigma^+}} \right) \right]$$

The second part of the sum can be obtained from the approximation to the GPD. Following Bollerslev and Todorov (2011b) I assume that for a large threshold value the following approximation holds with equality:

$$1 - \frac{\bar{v}_\psi^+(u+x)}{\bar{v}_\psi^+(x)} = G(u; \sigma^+, \xi^+)$$

where $G()$ denotes a GPD. Assuming that $x = tr^+$, $u = e^k - 1 - tr^+$ and $tr^+ = 0.075$, it is quite straight forward that:

$$\bar{v}_\psi^+(e^k - 1) = [1 - G(e^k - 1 - tr^+; \sigma^+, \xi^+)] \bar{v}_\psi^+(tr^+)$$

EQUITY PREMIUM DERIVATION

Appendix E - Filters

1. Removing contracts with zero time value
2. Removing contracts that violate convexity
3. Removing contracts that violate monotonicity
4. Removing options with identical delta
5. Removing options with delta [0.01 0.99]

Appendix F - Rolling Window correlations

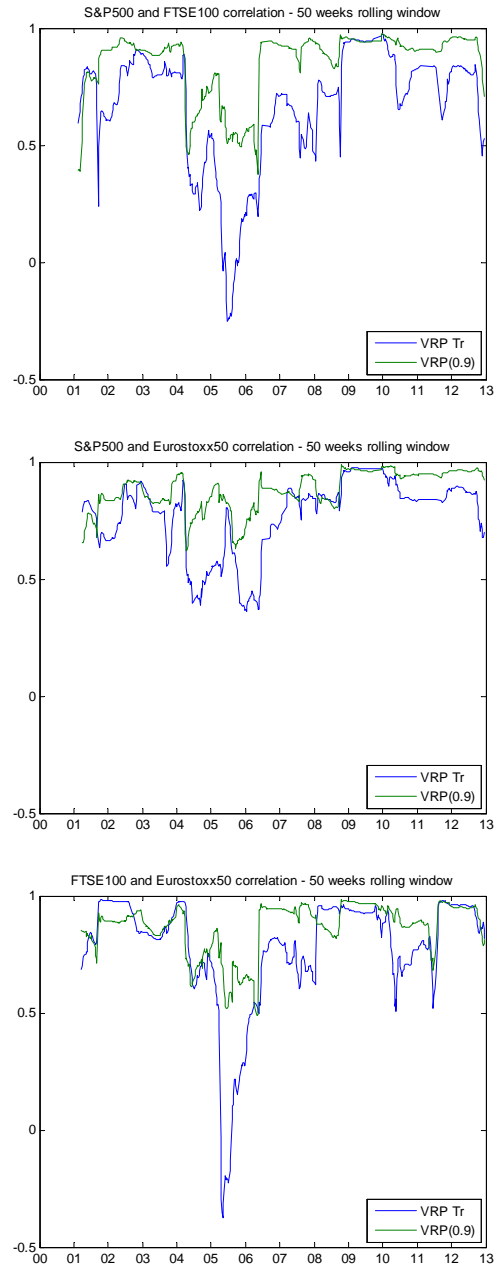


Figure 8: Time varying correlations of $VRP(Tr)$ and $VRP(0.9)$ for different pairs of indices. R-pearson coefficient calculated for the past 50 weekly observations.

Appendix G - Estimation results for VRP(0.9) and VRP Tr

Indices	Variable	Corr	λ_1	λ_2
S&P500; FTSE100	VRP Tr	02443 0.0651	0.0101 0.0071	0.9280 0.0878
	VRP(0.9)	0.7081 0.0978	0.0656 0.3243	0.5419 5.3576
S&P500; Eurostoxx50	VRP Tr	06462 0.1895	0.0290 0.0315	0.5449 0.2743
	VRP(0.9)	0.7034 0.1426	0.0101 0.0204	0.7318 0.1471
FTE100;Eurostoxx50	VRP Tr	0.6198 0.2476	0.0078 0.0230	0.8983 0.1007
	VRP(0.9)	0.7342 0.2000	0.0343 0.1010	0.6511 0.1404

Table 12: Dynamic Conditional Correlation coefficients

Appendix H - Investor Fear Indice correlation and both sided VRP Tr

	r-Pearson		Kendall τ	
	VRP(Tr-2)	FI	VRP(Tr-2)	FI
S&P500 FTSE100	0.8119	0.9364	0.4058	0.7385
S&P500 Eurostoxx50	0.8162	0.9199	0.4950	0.7279
FTSE100 Eurostoxx50	0.9413	0.9435	0.6192	0.7562

Table 13: Pairwise correlations of the VRP(Tr-2), truncated from both sides, and FI, based on the weekly data.

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