VOLATILITY, CORRELATION AND TAILS
FOR SYSTEMIC RISK MEASUREMENT

Christian T. Brownlees[*]   Robert Engle[*]

This Draft: June 2011

Abstract

In this paper we propose an empirical methodology to measure systemic risk. Building upon Acharya *et al.* (2010), we think of the systemic risk of a financial institution as its contribution to the total capital shortfall of the financial system that can be expected in a future crisis. We propose a systemic risk measure (SRISK) that captures the expected capital shortage of a firm given its degree of leverage and Marginal Expected Shortfall (MES). MES is the expected loss an equity investor in a financial firm would experience if the overall market declined substantially. To estimate MES, we introduce a dynamic model for the market and firm returns. The specification is characterized by time varying volatility and correlation, which are modelled with the familiar TARCH and DCC. We do not make specific distributional assumptions on the model innovations and rely on flexible methods for inference that allow for tail dependence. The model is extrapolated to estimate the equity loss of a firm in a future crisis and consequently the capital shortage that would be experienced depending on the initial leverage. The empirical application on a set of top U.S. financial firms finds that the methodology provides useful rankings of systemically risky firms at various stages of the financial crisis. One year and a half before the Lehman bankruptcy, eight companies out of the SRISK top ten turned out to be troubled institutions. Results also highlight the deterioration of the capitalization of the financial system starting from January 2007 and that as of July 2010 the system does not appear fully recovered.

Keywords: Systemic Risk, Volatility, Correlations, Tails, Forecasting.

JEL classification: C22, C23, C53.

1 Introduction

The Great Recession of 2007/2009 has motivated market participants, academics and regulators to better understand systemic risk. A useful definition of systemic risk by Federal Reserve Governor Daniel Tarullo, is

> *"Financial institutions are systemically important if the failure of the firm to meet its obligations to creditors and customers would have significant adverse consequences for the financial system and the broader economy."*

In this definition, it is the failure of a systemically important firm to meet obligations that is the cause of systemic distress. Thus, measures of systemic risk are associated with firm bankruptcies or near bankruptcies which are inevitable consequences of a decline in equity valuations for highly levered firms.

This idea is further developed in the theoretical analysis of Acharya *et al.* (2010). A financial firm is unable to function when the value of its equity falls to a sufficiently small fraction of its outstanding liabilities. In good times, such a firm is likely be acquired, may be able to raise new capital or may face an orderly bankruptcy. If this capital shortage occurs just when the financial sector is already financially constrained, then the government faces the question of whether to rescue the firm with taxpayer money as other avenues are no longer available. Such a capital shortage is damaging to the real economy as the failure of this firm will have repercussions throughout the financial and real sectors. Consequently a firm is systemically risky if it is likely to face a large capital shortfall just when the financial sector itself is under distress.

The goal of this paper is to propose an empirical methodology to measure capital shortage in order to produce a systemic risk index for an individual firm. The capital shortfall of a firm depends on its degree of leverage and the equity loss that would result from such a crisis. While the degree of leverage can be measured, the equity loss in a crisis must be predicted. We predict it with the Marginal Expected Shortfall (MES), the expected equity loss of a firm

2

when the overall market declines beyond a given threshold over a given time horizon. The capital shortfall of a firm in a crisis computed using leverage and MES is what determines our Systemic Risk index (SRISK). The companies with the highest SRISK are the companies that contribute the most to the market undercapitalization in a crisis and are therefore the most systemically risky firms. Conceptually this calculation is like the stress tests that are regularly applied to financial firms, however here it is done with only publicly available information and is quick and inexpensive.

In this work, we begin by reviewing the key elements of the Acharya *et al.* (2010) framework and introduce the SRISK index. In order to construct the MES predictions needed for the calculation of the index, we propose a conditionally heteroskedastic bivariate model for market and firm equity return dynamics. This model characterizes the behaviour of the returns in terms of time varying volatility, correlation and tails. The specification does not make specific distributional assumptions on the innovation term of the process. A multi step modeling approach based on GARCH and DCC (Engle (2002b), Engle (2009)) is used to fit volatility and correlations. Inference on the innovation term is based on flexible methods and allow for potential nonlinear dependence. The model is used to construct one and multi–period ahead MES forecasts, which we name, respectively, short and long run MES. The short and long run MES are defined, respectively, conditional on a 2% market drop over one day and 40% market drop over six months. In the short run case, predictions can be easily constructed using volatility, correlation predictions and nonparametric tail expectation estimators (Scaillet (2004), Scaillet (2005)). Long run MES forecasts cannot be obtained in closed form but simulation based methods yield such forecasts. The long run forecasts are the ones used in the construction of the SRISK index.

The proposed methodology is applied to analyse the systemic risk contribution of a sample of 94 top U.S. Financial firms between July 2000 and July 2010. Financial firms are grouped in 4 categories: Depositories, Insurance, Broker-Dealers and a residual category labelled Others which also contains non depository institutions and real estate related firms. We

3

begin by using our GARCH/DCC/Nonparametric approach to study the in sample dynamics of the panel. We then carry out two different prediction exercises. We first perform a short term MES forecasting application and use different criteria to compare the predictive ability of our approach against two benchmarks. We then engage a long run MES and SRISK forecasting exercise where we examine the rankings of systemically risky institution produced at various stages of the crisis as well as the dynamics of the aggregate SRISK in the sample.

The model implies that the most important determinants of MES are time varying firm volatility and correlation with the market. MES is high for firms that are volatile and are not diversified with respect to the market. The time series analysis reveals that the level of MES during the financial crisis is extreme by historical standards. However, the MES industry group rankings have been stable in time, with Broker-Dealers and the Other sectors being the most exposed ones. The short term MES forecasting application shows that our MES predictions are able to perform well relative to the benchmarks. The long term MES forecasting results show that SRISK delivers useful rankings of systemically risky firms at various stages of the financial crisis. One year and a half before the Lehman bankruptcy, eight companies out of the SRISK top ten turned out to be troubled institutions. Results also document the deterioration of the financial system capitalization starting from July 2007 and that as of July 2010 the financial system does not appear fully recovered yet.

Our work relates to a growing number of contributions on the analysis of the financial crisis and the measurement of systemic risk. A current challenge in this field of research is that there is no widespread agreement on how systemic risk ought to be defined and the measures being proposed have some degree of heterogeneity. The CoVaR of Adrian and Brunnermeier (2009) is one of the early proposal of the literature that has received notable attention. They relate systemic risk to the VaR of the market conditionally on an individual institution being under distress. Hautsch *et al.* (2010) define systemic risk as the time varying contribution of the firm VaR on the VaR of the market. Similarly, Manganelli *et al.* (2010) analyse spillover effects between the VaR of a financial institutions and the market. Billio *et al.* (2010) pro-

pose systemic risk indices based on Principal Component analysis and Granger–causality tests. Schwaab *et al.* (2011) introduce coincident risk measures and early warning indicators for financial distress which are extracted from macro and credit risk data. Contributions that relate to this area of research also include Allen *et al.* (2010), which assess the predictive power of aggregate systemic risk measures in predicting future downturns and Brownlees (2011), which proposes a model to relate financial firms dynamics to their characteristics. The methodology developed in this work builds upon the literature on volatility and correlation modeling using, respectively, GARCH and DCC models. A detailed glossary of the ARCH universe can be found in Bollerslev (2008). The DCC approach for correlations has been introduced by Engle (2002a) and recently surveyed in Engle (2009). Contributions in this area include Engle and Sheppard (2001), Aielli (2006) and Engle *et al.* (2009). Dynamic models for Value–at–Risk and Expected Shortfall have been developed in Engle and Manganelli (2004) and Taylor (2008). Nonparametric approaches to measures such expectations have been developed in Scaillet (2005).

The rest of the paper is organized as follows. Section 2 introduces the Acharya *et al.* (2010) framework and the SRISK index. Section 3 describes the econometric methodology used to in this work to estimate MES. Section 4 illustrates the empirical application on a set of top U.S. financial firms. Concluding remarks follow in Section 5. The methology developed in this paper is used to analyse top U.S. financials on a weekly basis on the systemic risk section of the Vlab (http://vlab.stern.nyu.edu/).

2 Measuring Systemic Risk

Acharya *et al.* (2010) develop a two period model where a financial institution i chooses in period 1 how much capital to raise from risky debt, F_i, guaranteed debt, G_i, and initial capital, W_i. This is then invested in J assets by taking exposures $X_{i1}, ..., X_{iJ}$. The risky debt is priced at a discount rate B_1 and the guaranteed debt is for simplicity priced at par. In

period 2, the exposures pay total return $r_1, ..., r_J$ and the debt must be repaid at face value. Because of bankruptcy costs and costs of capital shortage in general, there may be additional costs in period 2. Thus, the budget constraint in period 1 is

$$W_{i1} + F_{i1}B_1 + G_{i1} = \sum_{j=1}^{J} X_{ij}.$$

The firm in period 1 has to choose investments X and borrowing F from long term capital markets. Additional funds can come from negative X's which would include short term roll–over financing. The firm chooses leverage to maximise returns based upon the distribution of equity value in the second period and the risk of bankruptcy. In period 2 the net worth of the institution is

$$W_{i2} = \sum_{j=1}^{J} X_{ij}r_j - G_{i1} - F_{i1} - \phi,$$

where ϕ is the cost of distress which could be bankruptcy or it could be the failure to carry out plans because of capital shortage. In general it will be a function of the state variables. If W_{i2} is smaller than zero then the firm is insolvent. If W_{i2} is positive yet small the firm experiences a capital shortage and might be unable to function properly.

The key assumption of Acharya *et al.* (2010) is that capital shortages of an individual firm impose external costs on the real economy when they occur during a period of distress for the whole system. These costs are first of all costs to taxpayers who offer the guarantee in G_{i1}. They however also include externalities that are particularly severe when the entire financial sector is capital constrained. When the economy is in a downturn, the bankruptcy of a firm cannot be absorbed by a stronger competitor and obligations will spread throughout both the financial and real economy as the natural functions of the financial sector will be curtailed. When the system is undercapitalized, it will no longer supply credit for ordinary everyday business and the economy will grind to a halt. Thus the shortage of capital is dangerous for one firm and for its bondholders, but it is dangerous for the economy if it occurs just when

the rest of the sector is undercapitalized. Note that the objective function of the firm does not take into account the negative externality costs that it generates in a crisis. Hence, firms can take excessive leverage in the absence of regulatory intervention. When the volatility of returns is low for instance, risk is low and the optimal leverage is high.

It is of interest to measure what the capital shortfall would be for each firm in a crisis. If a prudential ratio of asset value to equity is k, then the firm's capital buffer at the end of period 1 is

$$k(B_1 F_{i1} + G_{i1} + W_{i1}) - W_{i1},$$

and when this value is positive the firm is said to experience a capital shortage. We then estimate the expected equity loss in a crisis and compute the consequent capital shortfall. We define a crisis as a drop of the market index below a certain threshold C, which we call for short "systemic event". The expected capital shortfall CS_{i1} in period 2 is estimated in period 1 to be

$$
\begin{aligned}
\mathsf{CS}_{i1} &= \mathsf{E}_1(k(F_{i1} + G_{i1} + W_{i2}) - W_{i2}|\mathsf{Crisis}) \\
&= k(F_{i1} + G_{i1}) - (1-k)\,\mathsf{E}_1(W_{i2}|\mathsf{Crisis}) \\
&= k(F_{i1} + G_{i1}) - (1-k)\,W_{i1}\,\mathsf{E}_1(R_{i2}|R_{m2} < C) \\
&= k(F_{i1} + G_{i1}) - (1-k)\,W_{i1}\,\mathsf{MES}_{i1}
\end{aligned}
$$

where R_{i2} and R_{m2} are the firm and market (arithmetic) returns in period 2 and MES stands for Marginal Expected Shortfall, the tail expectation of the firm return conditional on the market being in its left tail.

A large capital shortfall in the financial system will cause a real crisis. The firms with the largest capital shortfall are the greatest contributors to the crisis and are the institutions considered as most systemically risky. Hence, large expected capital shortage in a crisis does not just capture individual firm vulnerability but also systemic risk. We define the systemic

risk index of institution i as

$$\mathsf{SRISK}_i = \min(0, \mathsf{CS}_{i\,0}),$$

and its percentage version as

$$\mathsf{SRISK}\%_i = \mathsf{SRISK}_i / \sum_i \mathsf{SRISK}_i.$$

Hence, the $\mathsf{SRISK}\%$ index measures the portion of the total expected system capital shortfall in a crisis that is due to firm i.

In practice, our SRISK index has different determinants. An increase in the size of the firm, keeping the leverage ratio constant, increases systemic risk in that it makes the capital shortage potentially wider. An increase in debt also has positive impact on systemic risk in that it makes the capital buffer of the firm higher as well. Finally, a high downside exposure of the firm to systematic shocks in the economy contributes positively to an increase of systemic risk.

An appealing feature of the SRISK index is that it can be constructed using publicly available data, making the index widely applicable and relatively inexpensive to implement.

3   Econometric Methodology

The computation of the systemic risk indices requires knowledge of the debt, equity and MES of each firm. While debt and equity values are readily available, the estimation of the MES requires the introduction of appropriate time series techniques. In this work we are interested in estimating the capital shortfall over a potentially long time period like (say, a quarter or six months) and we need time series methods able to deliver estimates of MES over long horizons. Many strategies can be devised to attack this problem. In this work we

tackle the issue by specifying a bivariate conditionally heteroskedastic model to characterize the dynamics of the daily firm and market returns. Once the model is estimated, the process can be easily extrapolated to produce the MES predictions of interest.

Let r_{it} and r_{mt} denote respectively firm's i and the market log return on day t. Our approach starts from a description of the bivariate process of the firm and market returns:

$$
\begin{aligned}
r_{mt} &= \sigma_{mt}\,\epsilon_{mt} \\
r_{it} &= \sigma_{it}\,\rho_{it}\epsilon_{mt} + \sigma_{it}\sqrt{1-\rho_{it}^2}\xi_{it} \\
(\epsilon_{mt},\xi_{it}) &\sim F
\end{aligned}
$$

where the shocks (ϵ_{mt},ξ_{it}) are independent and identically distributed over time and have zero mean, unit variance and zero covariance. However they are not assumed to be independent. Indeed, there are important reasons to believe that extreme values of these disturbances could occur at the same time for systemically risky firms. When the market is in its tail, the firm disturbances may be even further in the tail if there is serious risk of default.

The stochastic specification is completed by a description of the two conditional standard deviations and the conditional correlation. These will be discussed in the next section but are familiar models of TARCH and DCC. On the other hand, the distribution F is left unspecified and we will adopt flexible nonparametric approach for inference.

The one period ahead MES can be straightforwardly expressed as a function of volatility,

correlation and tail expectations of the standardised innovations distribution

$$
\begin{aligned}
\mathsf{MES}^1_{i\,t-1}(C) &= \mathsf{E}_{t-1}(r_{it}|r_{mt} < C) \\
&= \sigma_{it}\mathsf{E}_{t-1}(\epsilon_{it}|\epsilon_{mt} < C/\sigma_{mt}) \\
&= \sigma_{it}\mathsf{E}_{t-1}(\rho_t\epsilon_{mt} + \sqrt{1 - \rho_t^2}\xi_{it}|\epsilon_{mt} < C/\sigma_{mt}) \\
&= \sigma_{it}\rho_t\mathsf{E}_{t-1}(\epsilon_{mt}|\epsilon_{mt} < C/\sigma_{mt}) + \\
&\quad \sigma_{it}\sqrt{1 - \rho_t^2}\mathsf{E}_{t-1}(\xi_{it}|\epsilon_{mt} < C/\sigma_{mt}),
\end{aligned}
\tag{1}
$$

and the conditional probability of a systemic event is

$$
\mathsf{PoS}^1_t(C) = P_{t-1}(r_{mt} < C) = P(\epsilon_{mt} < C/\sigma_{mt}).
\tag{2}
$$

Some comments on the formulas in Equations (1) and (2) are in order under the assumption that the dependence between the firm and the market is positive. Firstly MES is an increasing function of a firm's own volatility. Depending on whether correlation is high or low, the MES formula gives more weight to, respectively, either the tail expectation of the standardised market residual or the tail expectation of standardised idiosyncratic firm residual. The second term in Equation (1) arises because of the nonlinear dependence assumption between ϵ_{mt} and ξ_{it} and it would otherwise be zero if dependence was captured entirely by correlation. Secondly, MES relates to the systematic risk "beta" of the CAPM. If the data are generated by a one factor model then MES is equal to systematic risk times the Expected Shortfall of the market. Our approach however is more flexible in that it allows for time varying moments and focuses on downside exposure. It is also important to stress the implication of the conditioning systemic event C. Typically, VaR and ES are expressed in conditional terms, that is the conditioning event is a quantile from the conditional return distribution. On the other hand, in this work the conditioning event is unconditional. Thus, while in the conventional approach the probability of observing the conditioning event is constant, in our

framework such probability is time varying: the higher the volatility the higher the probability of observing a loss above a given threshold. Equation (1) has an approximation error due to the fact we are using log returns rather than arithmetic returns. In this work we do not add an adjustment term to the formula, but this has been worked out in Caporin and De Magistris (2011). In what follows we are going to call the 1 period ahead MES as "short term" and we are going to set the constant C to a 2% market drop.

The multi–period ahead MES cannot be obtained in closed form and a simulation procedure is used to construct the forecasts. The procedure is designed as follows. In order to produce the h period MES starting from day t, on day $t-1$ we simulate S return paths of length h

$$\left\{ \begin{array}{c} r^s_{m\,t+\tau-1} \\ r^s_{i\,t+\tau-1} \end{array} \right\}^h_{\tau=1} \qquad s = 1, ..., S.$$

We obtain the paths by first drawing pseudo innovations from the innovation distribution F, that is

$$(\epsilon^s_{m\,t+\tau-1}, \xi^s_{m\,t+\tau-1})^h_{\tau=1} \sim F,$$

and by then "re-colouring" them through the DCC and GARCH filters using the current levels of volatility and correlation as starting conditions. The multi period MES is calculated using the Monte Carlo average of the simulated paths,

$$\mathsf{MES}^h_{i\,t-1}(C) = \frac{\sum^S_{s=1} R^s_{i\,t:t+h-1} I\{R^s_{m\,t:t+h-1} < C\}}{\sum^S_{s=1} I\{R^s_{m\,t:t+h-1} < C\}}, \tag{3}$$

where $R^s_{i\,t:t+h-1}$ denotes the s simulated cumulative return of firm i from period t to period $t+h-1$, i.e.

$$R^s_{i\,t:t+h-1} = \exp\left\{ \sum^h_{\tau=1} r^s_{i\,t+\tau-1} \right\} - 1, \tag{4}$$

and analogously for the market return $R^s_{m\,t:t+h-1}$. Analogously, the multi–period probability

11

of a crisis is given by

$$\mathsf{PoS}_t^h(C) = P_t(R_{m\,t:t+h-1} < C) = \frac{1}{S}\sum_{s=1}^{S} I\{R_{m\,t:t+h-1}^s < C\}.$$

This approach is close in spirit to what risk management practitioners call "scenario analysis", that is extrapolating the risk implied by the model conditionally on a stream of adverse outcomes. In what follows we are going to consider a 6 months period MES using a market drop C of 40% and we are going to call this prediction long run MES. A useful feature of long run forecasts is that they partly counter the pro–cyclical nature of short term risk assessment. Averaging over long sequences of events in which the market has a substantial fall allows us to evaluate the model implied downside exposure of a firm while giving less importance to the idiosyncrasy of current market conditions.

Different strategies can be employed to obtain estimates of MES at the horizons of interest. In this work we rely on a multi stage modeling approach which is inspired by the DCC (Engle (2002a), Engle (2009)). In the first step we model volatilities using GARCH models to obtain conditional volatility and standardised residuals. We then resort to a DCC specification to obtain conditional correlation and the standardised idiosyncratic firm residual. Finally, inference on the model innovations is based on the GARCH/DCC residuals. The appealing features of such a modelling paradigm are simplicity and flexibility. Estimation of a fully bivariate conditionally heteroskedastic model with nonlinear residual dependence for a large panel of assets can be quite challenging, especially when the time series are not too long. On the other hand our approach is much easier to implement and it allows for considerable flexibility in the specifications, by changing the different types of volatility (splines, high frequency based, etc.) and correlation (standard DCC, factor DCC, asymmetric, breaks in correlation, etc.).

In what follows we make the follow modelling choices:

Volatility. In the wide universe of GARCH specifications, we pick the TARCH specification to model volatility (Rabemananjara and Zakoïan (1993), Glosten *et al.* (1993)). The evolution of the conditional variance dynamics in this model class is given by

$$\sigma_{mt}^2 = \omega_{mG} + \alpha_{mG}\, r_{mt-1}^2 + \gamma_{mG}\, r_{mt-1}^2 I_{mt-1}^- + \beta_{mG}\, \sigma_{mt-1}^2$$

$$\sigma_{it}^2 = \omega_{iG} + \alpha_{iG}\, r_{it-1}^2 + \gamma_{iG}\, r_{it-1}^2 I_{it-1}^- + \beta_{iG}\, \sigma_{it-1}^2$$

with $I_{it}^- = r_{it} < 0$ and $I_{mt}^- = r_{mt} < 0$. The main highlight of this specification is its ability to capture the so called leverage effect, that is the tendency of volatility to increase more with negative news rather than positive ones. This model is also successful from a forecasting standpoint and it turns out to be quite difficult to beat. We estimate the model using QML which guarantees consistent estimates of the model parameters as long as the conditional variance equation is correctly specified.

Correlations. We model time varying correlations using the DCC approach (Engle (2002a), Engle (2009)). Let P_t denote the time varying correlation matrix of the market and firm return, that is, using matrix notation,

$$\mathsf{Var}_{t-1}\begin{pmatrix} r_{it} \\ r_{mt} \end{pmatrix} = D_t\, P_t\, D_t$$

$$= \begin{bmatrix} \sigma_{it} & 0 \\ 0 & \sigma_{mt} \end{bmatrix} \begin{bmatrix} 1 & \rho_t \\ \rho_t & 1 \end{bmatrix} \begin{bmatrix} \sigma_{it} & 0 \\ 0 & \sigma_{mt} \end{bmatrix}.$$

Rather than directly modeling the P_t matrix, the DCC framework models the so–called pseudo correlation matrix Q_t, a positive definite matrix which is then mapped in a correlation matrix through the transformation

$$P_t = \mathsf{diag}(Q_t)^{-1/2}\, Q_t\, \mathsf{diag}(Q_t)^{-1/2},$$

where the $\text{diag}(A)$ matrix operator denotes a matrix with the same elements of the A matrix on the diagonal and zero otherwise. We formulate the pseudo correlation matrix Q_t dynamics using the DCC formulation proposed in Aielli (2006) and Aielli (2009).

The basic (scalar) symmetric DCC specification is defined as

$$Q_t = (1 - \alpha_C - \beta_C)S + \alpha_C \, \epsilon^*_{t-1}\epsilon^{*\,'}_{t-1} + \beta_C \, Q_{t-1}, \tag{5}$$

where S is an intercept matrix and ϵ^*_t contains the rescaled standardised (or degarched) returns, that is $\epsilon^*_t = Q^*_{t-1}\epsilon_t$ with $Q^*_t = \text{diag}(Q_t)^{1/2}$. The pseudo conditional correlation matrix Q_t is thus an exponentially weighted moving average of past outer products of the rescaled standardised returns. Necessary and sufficient conditions for Q_t to be positive definite are $\alpha_C > 0$, $\beta_C > 0$, $\alpha_C + \beta_C < 1$ and the positive definiteness of the S matrix. The rescaling device ensures that $\{\epsilon^*_t, Q_t\}$ is a MGARCH process (Ding and Engle (2001)) and, under the assumption of stationarity of the model $(\alpha_C + \beta_C < 1)$, this implies that S is the unconditional covariance matrix of ϵ^*_t

$$S = \mathsf{E}(\epsilon^*_t \epsilon^{*\,'}_t).$$

This property is useful for highly dimensional DCC estimation in that it justifies the use of the unconditional covariance matrix of the ϵ^*_t as a correlation targeting (Mezrich and Engle (1996)) estimator for S, that is

$$\hat{S} = \frac{1}{n}\sum \epsilon^*_t \epsilon^{*\,'}_t,$$

which drastically reduces the number of parameter that need to be optimized to estimate the model. A slight complication of the $\hat{S}$ estimator is that it is based on the diagonal of the Q_t matrix which in turns depends on the parameter values of the model. Hence, in the evaluation of the likelihood, for instance, the estimators have to be computed each time. Also note that the diagonal elements of S are either known or can be computed without knowledge of the model parameters. The computation of the likelihood proceeds by first computing the

14

diagonal of Q_t, evaluating the $\hat{S}$ matrix and finally computing the off diagonal elements of Q_t and the likelihood of the model.

Tail Expectations for Short Term MES. To compute short term MES in Equation (1) we need to estimate of the tail expectations

$$\mathsf{E}(\epsilon_{mt}|\epsilon_{mt} < \kappa) \qquad \text{and} \qquad \mathsf{E}(\xi_{it}|\epsilon_{mt} < \kappa).$$

These expectation can be simply estimated for a particular value of the variances $(\sigma^2_{mt}, \sigma^2_{it})$ and conditional correlation ρ_t by simply looking at the average of the two residuals in all cases which satisfy the condition $\epsilon_{mt} < \kappa$. However, when $-\kappa$ is large, this estimator will be unstable as there are only a small number of observations. A nonparametric kernel estimation approach can be used to improved the efficiency of these simple estimators. Let

$$K_h(t) = \int_\infty^{t/h} k(u)du,$$

where $k(u)$ is a kernel function and h is a positive bandwidth. Then

$$\widehat{\mathsf{E}}_h(\epsilon_{mt}|\epsilon_{mt} < \kappa) = \frac{\sum_{i=1}^n \epsilon_{mt} K_h(\epsilon_{mt} - \kappa)}{(n\hat{p}_h)}, \tag{6}$$

and

$$\widehat{\mathsf{E}}_h(\xi_{it}|\epsilon_{mt} < \kappa) = \frac{\sum_{i=1}^n \xi_{it} K_h(\epsilon_{mt} - \kappa)}{(n\hat{p}_h)}, \tag{7}$$

where

$$\hat{p}_h = \frac{\sum_{i=1}^n K_h(\epsilon_{mt} - \kappa)}{n}.$$

An advantage of the nonparametric estimators defined in Equations (6) and (7) is that they are smooth functions of the cutoff point κ which, in turns, deliver smooth estimates of short

term MES as a function of κ. The properties of these type of nonparametric tail expectation estimators are discussed in Scaillet (2005).

Simulations for Long Term MES. To compute the long term MES of Equation (3), we need to draw samples from the innovation distribution F. The sampling strategy we adopt is to sample with replacement from the empirical cumulative density function of the estimated residuals $\widehat{F}$. A number of algorithmic shortcuts can be implemented to substantially reduce the computational burden of the long term MES computation in large panel of firms. The strategy we adopt is to simulate S market paths and check which paths meet the systemic event condition. For each of these paths, we store the sequence of id's of the selected draws. Then, the simulation of the individual firm trajectories consists of constructing for each selected path the sequence of individual firm shocks which correspond to those of the market. This speeds up the simulations in that it avoids having to simulate and select paths for each firm/market return pair in the panel.

4 Empirical Analysis

4.1 Data

We study the same panel of institutions studied in Acharya *et al.* (2010) between July 3, 2000 and June 30, 2010. The panel contains all U.S. financial firms with a market capitalization greater than 5 bln USD as of the end of June 2007 and it is unbalanced in that not all companies have continuously been trading during the sample period. We extract daily returns and market capitalization from CRSP and the quarterly book value of equity (ceqq) from COMPUSTAT. SIC codes are used to divide firms into 4 groups: Depositories (such as Bank of America or JP Morgan Chase), Broker-Dealers (Goldman Sachs or Lehman Brothers), Insurance (AIG) and Others (non depositary institutions, real estate) (Freddie and Fannie). We make one exception to this rule, Goldman Sachs (GS) should have been classified within the

Depositories (29)		Insurance (32)		Broker-Dealers (10)		Others (23)	
BAC	Bank of America	ABK	Ambac Financial Group	AGE	A.G. Edwards	ACAS	American Capital
BBT	BB&T	AET	Aetna	BSC	Bear Stearns	AMP	Ameriprise Financial
BK	Bank of New York Mellon	AFL	Aflac	ETFC	E-Trade Financial	AMTD	TD Ameritrade
C	Citigroup	AIG	American International Group	GS	Goldman Sachs	AXP	American Express
CBH	Commerce Bancorp	AIZ	Assurant	LEH	Lehman Brothers	BEN	Franklin Resources
CMA	Comerica inc	ALL	Allstate Corp	MER	Merrill Lynch	BLK	Blackrock
HBAN	Huntington Bancshares	AOC	Aon Corp	MS	Morgan Stanley	BOT	CBOT Holdings
HCBK	Hudson City Bancorp	WRB	W.R. Berkley Corp	NMX	Nymex Holdings	CBG	C.B. Richard Ellis Group
JPM	JP Morgan Chase	BRK	Berkshire Hathaway	SCHW	Schwab Charles	CBSS	Compass Bancshares
KEY	Keycorp	CB	Chubb Corp	TROW	T. Rowe Price	CIT	CIT Group
MI	Marshall & Ilsley	CFC	Countrywide Financial			CME	CME Group
MTB	M & T Bank Corp	CI	CIGNA Corp			COF	Capital One Financial
NCC	National City Corp	CINF	Cincinnati Financial Corp			EV	Eaton Vance
NTRS	Northern Trust	CNA	CNA Financial corp			FITB	Fifth Third Bancorp
NYB	New York Community Bancorp	CVH	Coventry Health Care			FNM	Fannie Mae
PBCT	Peoples United Financial	FNF	Fidelity National Financial			FRE	Freddie Mac
PNC	PNC Financial Services	GNW	Genworth Financial			HRB	H&R Block
RF	Regions Financial	HIG	Hartford Financial Group			ICE	Intercontinental Exchange
SNV	Synovus Financial	HNT	Health Net			JNS	Janus Capital
SOV	Sovereign Bancorp	HUM	Humana			LM	Legg Mason
STI	Suntrust Banks	LNC	Lincoln National			NYX	NYSE Euronext
STT	State Street	MBI	MBIA			SEIC	SEI Investments Company
UB	Unionbancal Corp	MET	Metlife			SLM	SLM Corp
USB	US Bancorp	MMC	Marsh & McLennan				
WB	Wachovia	PFG	Principal Financial Group				
WFC	Wells Fargo & Co	PGR	Progressive				
WM	Washington Mutual	PRU	Prudential Financial				
WU	Western Union	SAF	Safeco				
ZION	Zion	TMK	Torchmark				
		TRV	Travelers				
		UNH	Unitedhealth Group				
		UNM	Unum Group				

Table 1: Tickers, company names, industry groups.

		Avg. Ret.	Vol.	Corr.	Beta	QLVG
		Pre-Crisis (2005-07 to 2007-07)				
Depos.	Q_1	0.01	14.85	0.53	0.86	4.92
	$Median$	0.03	15.95	0.60	0.93	5.90
	Q_3	0.08	17.80	0.66	1.00	6.86
Insur.	Q_1	0.06	15.56	0.36	0.76	2.45
	$Median$	0.12	18.10	0.50	0.83	3.08
	Q_3	0.17	24.88	0.57	1.00	6.83
Bro.-Deal.	Q_1	0.21	20.81	0.62	1.39	1.67
	$Median$	0.23	23.43	0.69	1.55	8.55
	Q_3	0.30	27.84	0.74	1.67	13.05
Other	Q_1	0.00	19.66	0.42	0.96	1.14
	$Median$	0.17	24.20	0.49	1.21	1.51
	Q_3	0.31	31.13	0.56	1.49	5.26
		Crisis (2007-07 to 2009-07)				
Depos.	Q_1	-0.98	67.08	0.60	1.37	7.20
	$Median$	-0.37	92.44	0.67	1.78	13.02
	Q_3	-0.18	112.31	0.71	2.09	18.22
Insur.	Q_1	-0.58	55.57	0.56	0.99	2.40
	$Median$	-0.37	68.86	0.65	1.33	4.19
	Q_3	-0.24	105.06	0.73	2.11	19.60
Bro.-Deal.	Q_1	-1.39	65.32	0.52	1.47	1.46
	$Median$	-0.41	86.48	0.76	1.87	20.76
	Q_3	-0.11	136.69	0.77	2.33	39.10
Other	Q_1	-0.70	64.89	0.61	1.45	1.73
	$Median$	-0.45	78.32	0.72	1.76	2.29
	Q_3	-0.24	113.40	0.80	2.06	11.48
		Post-Crisis (2009-07 to 2010-07)				
Depos.	Q_1	-0.10	32.33	0.58	1.17	7.31
	$Median$	0.24	37.18	0.64	1.38	10.84
	Q_3	0.38	52.62	0.69	1.62	15.93
Insur.	Q_1	0.12	25.05	0.48	0.84	2.45
	$Median$	0.22	36.45	0.70	1.14	4.48
	Q_3	0.33	43.50	0.80	1.82	17.44
Bro.-Deal.	Q_1	-0.21	33.00	0.61	1.12	3.36
	$Median$	-0.14	34.14	0.66	1.40	10.41
	Q_3	-0.07	42.10	0.76	1.56	18.44
Other	Q_1	-0.11	32.48	0.64	1.15	1.73
	$Median$	0.04	37.96	0.71	1.43	2.24
	Q_3	0.40	47.08	0.79	1.76	7.16

Table 2: Descriptive statistics.

Others group, but instead we put it with Brokers-Dealers. We also use the daily CRSP market value weighted index return as the market index return. The full list of tickers and company names divided by industry groups is reported in Table 1.

Figure 1 gives visual insights on the boom and bust of the financial sector. The figure shows the cumulative average return by industry group from July 2007 to July 2010. Between July 2005 and June 2007 all financial groups had a steep growth, which is particularly strong for the Broker-Dealer group. Starting from July 2007, the fall of financials has been dramatic, with the biggest winners transforming into the biggest losers. Financials hit the bottom in
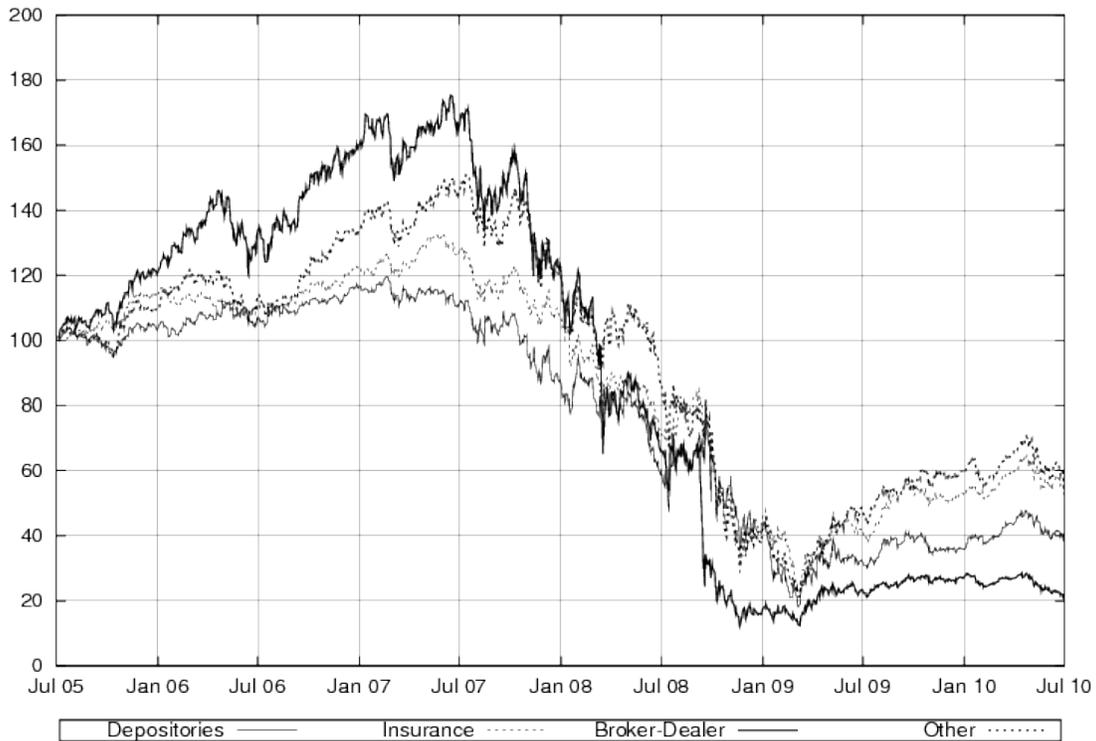
Figure 1: Cumulative average return by industry group.

March 2010 and start a slow recovery that is then interrupted by the European crisis of May 2010.

Table 2 reports descriptive statistics of the set of tickers divided by industry group over three subsamples (the pre-crisis, crisis and post-crisis). For each statistic the table reports the median, 1st and 3rd quartiles across each group.

4.2   Full Sample Estimation Results

We use the methodology introduced in Section 3 to analyse the panel. TARCH and DCC models are fitted on each firm over the whole sample period and in this section we report summary information on the parameter estimates, fitted series and residual diagnostics.

The dynamics of the firms in the panel do not have a strong degree of heterogeneity. In Table 3 we show selected quantiles of the parameter estimates of the TARCH (left side) and

19

		vol	α_V	γ_V	β_V	cor	α_C	β_C
Depos.	Q_1	32.21	0.02	0.08	0.87	0.55	0.03	0.89
	$Median$	36.92	0.04	0.10	0.91	0.62	0.04	0.93
	Q_3	40.14	0.07	0.12	0.92	0.66	0.05	0.96
Insur.	Q_1	29.93	0.02	0.09	0.86	0.44	0.01	0.91
	$Median$	37.64	0.04	0.11	0.90	0.52	0.03	0.95
	Q_3	44.12	0.05	0.13	0.92	0.58	0.05	0.98
Bro.–Deal.	Q_1	37.86	0.00	0.09	0.89	0.65	0.01	0.92
	$Median$	41.55	0.01	0.12	0.93	0.69	0.03	0.95
	Q_3	45.99	0.03	0.22	0.95	0.73	0.04	0.97
Other	Q_1	36.03	0.02	0.07	0.87	0.47	0.01	0.92
	$Median$	42.57	0.04	0.09	0.90	0.58	0.02	0.96
	Q_3	49.84	0.05	0.11	0.92	0.65	0.04	0.98

Table 3: TARCH and DCC estimation results.

DCC (right side) models for each group. The TARCH parameters do not fluctuate much, with the exception of the intercept which is on average higher for Broker-Dealers and Others. The range of the asymmetric coefficient γ reaches more extreme values for Broker-Dealers, signaling higher sensitivy to large volatility increases in case of a drop of the stock. Over ally, the point estimates are in line with the typical TARCH estimates, with slightly higher α's and γ's together with lower β's implying a higher level of unconditional kurtosis. Turning to the DCC, parameters are again close to the typical set of estimates and, intercept aside, parameters are similar across groups. Broker-Dealers have the highest level of unconditional correlation, followed by Others, Insurance and Depositories.

Figure 2 displays average volatility by industry group between 2005 and 2010. The plots are dominated by the explosion in variability at the peak of the crisis, which is so prominent it visually dominates all other periods of distress in the sample. The Depositories and Broker–Dealer series have extreme volatility spikes corresponding to well known events in the financial crisis time line: Depositories jump over 200% with the bankruptcy of Washington Mutual (September 2008) and Broker-Dealers go beyond 200% with the acquisition of Bears Sterns (March 2008) and the bankruptcy of Lehman (September 2008). Over all, all groups exhibit a similar time series trend which is also the one of market volatility. The pre crisis period is characterised by extremely low levels of variability. Starting from July 2007,
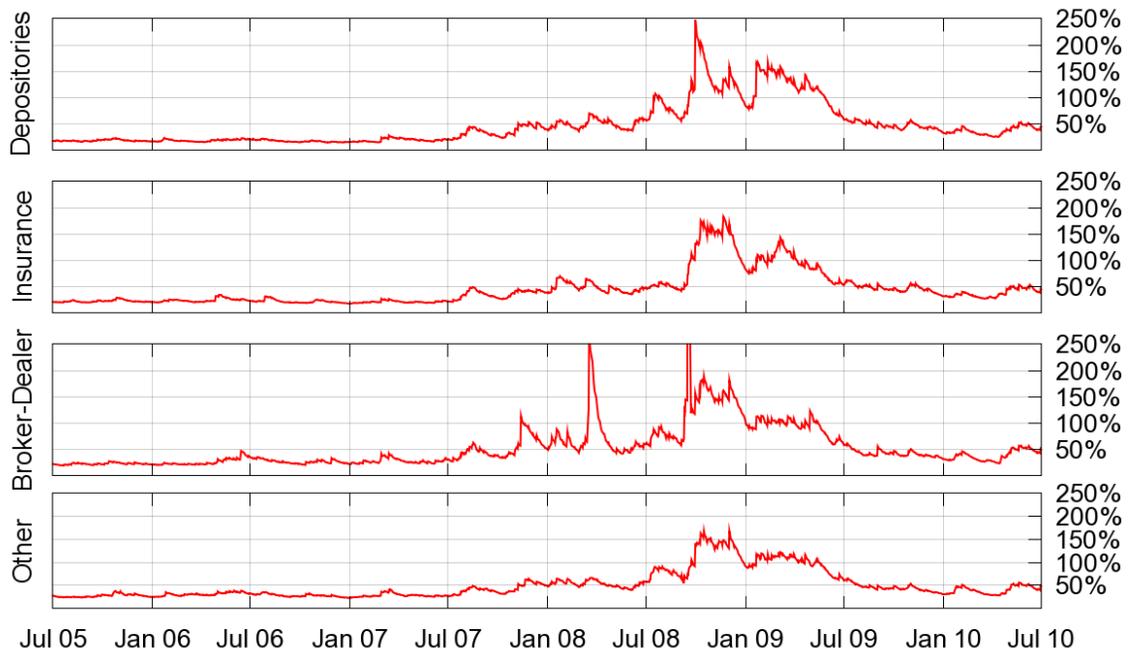
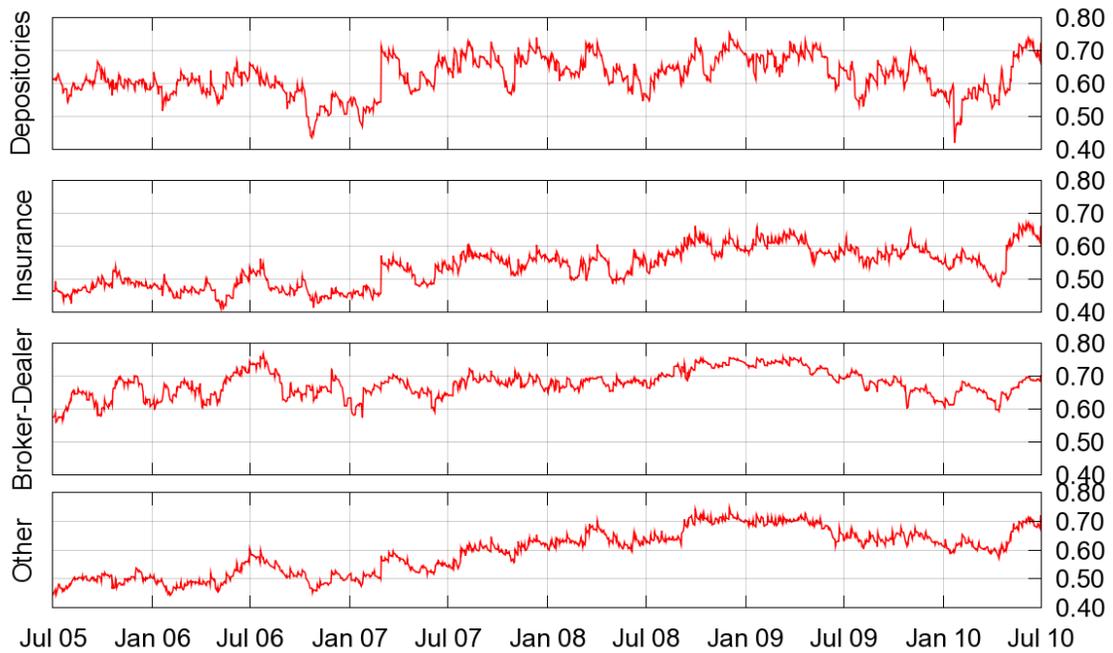Figure 2: Average volatility by financial industry group.

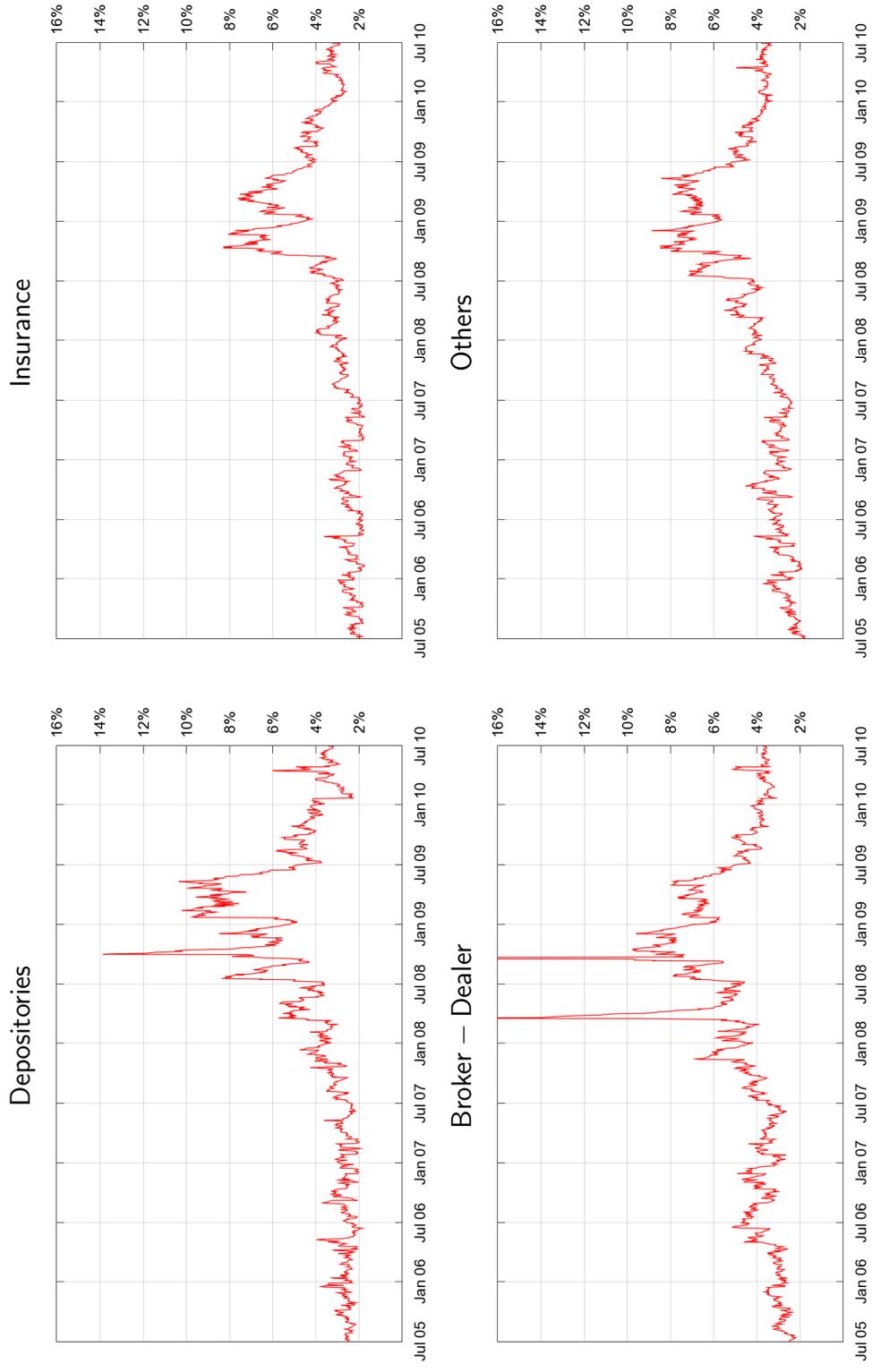Figure 3: Average correlation by financial industry group.
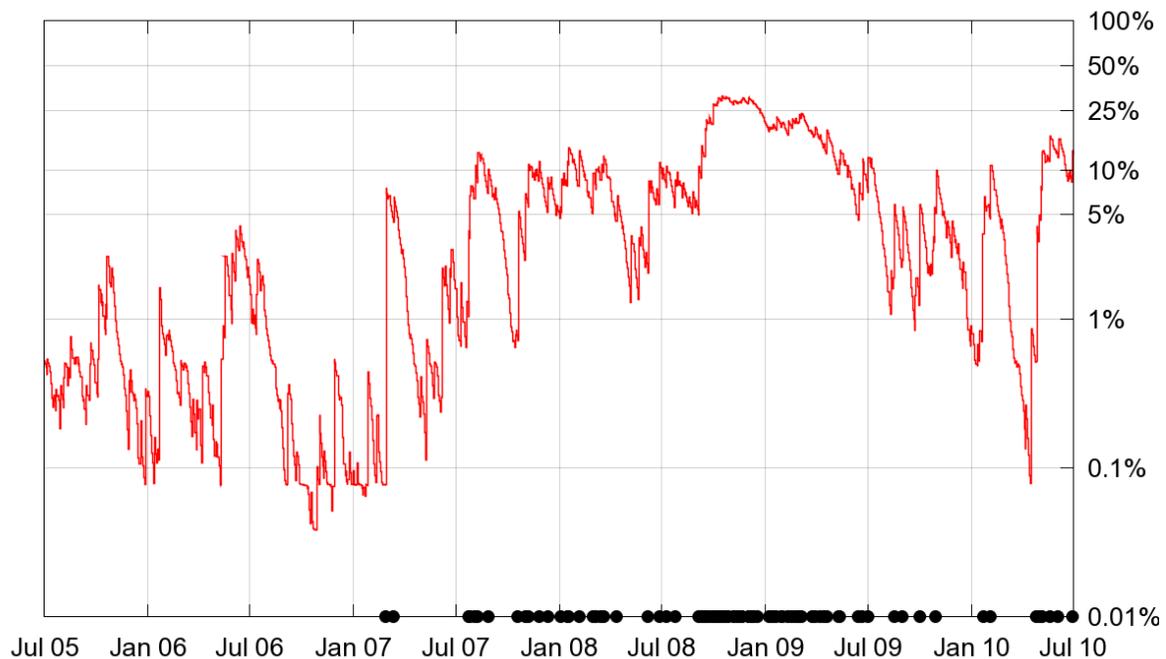
Figure 4: Average MES by financial industry group.

Figure 5: 1-step-ahead probability of a daily 2% loss in the market.

volatility gradually increases as the financial crisis unwraps and in October 2008, after the Lehman bankruptcy, it peaks to the highest levels ever measured since the 1987 crash. It then slowly decays and by March 2009, when the market recovery begins, its level is roughly the same as of 2007. All groups progressively cool down until May 2010, when news concerning the European crisis produce a new, yet moderate, surge in volatility. While the overall volatility trend is similar across groups the main difference between series is their average level. Broker-Dealers together with Other are most volatile ones followed by Depositories and Insurance.

Figure 3 shows average correlation by industry group. Once again, the time series pattern is similar although there are some differences between Broker-Dealers and the other three groups. Interestingly, while volatilities before the crisis are low correlations are moderately high by historical standards, ranging between 0.40 and 0.70. The so–called Chinese correction of February 29, further shifts correlations upwards, with the largest increase for Depos-

	Avg. Hit. Violation	10% Reject. Prop
Depos.	0.01310	0.31034
Insur.	0.01277	0.37500
Bro.-Deal.	0.01201	0.30000
Other	0.01213	0.08696

Table 4: Residuals Diagnostics.

itories. As the financial crisis unwinds correlations continue levitating, with Depositories and Broker-Dealers exceeding 0.75 in December 2008. In the post crisis period, correlations slowly begin to decrease until May 2010 when there is a steep increase caused by the European crisis which parallels the surge of volatility. As for volatility, the average levels of correlation differ across groups. Broker-Dealers have the highest correlation, Insurance the lowest one and Depositories together with the Other group lie in the middle.

Figure 4 reports the average short term MES by industry group. In the pre crisis period, the levels of MES appear to be roughly similar. The series start to increase from July 2007, reaching their peaks in October 2008. Depositories reach a MES of roughly 7% and go beyond 10% at the end of September 2008, corresponding to the bankruptcy of Washington Mutual. Broker-Dealers on the other hand have the biggest increase in mid March 2008 and mid September 2008 with the acquisition of Bears Sterns and liquidation of Lehman. MES starts to drop only after March 2009 and by January 2010 is back to its pre crisis levels. The European crisis does increase the level of MES but its impact is moderate. Once more, the main difference across series lies in their average levels. Broker-Dealers are the most exposed group followed by the Others, Depositories and Insurance. We also report in Figure 5 the time varying probability of observing the systemic event $\{r_{mt} < C\}$. The rug plot displays the actual days on which a systemic event is observed. The probability of a systemic event is a function of the market volatility and. as it can be readilliy notices, this series has essentially the same time series profile.

The inspection of residual diagnostics shows that the specification adequately fits the

	TARCH/DCC/NP	Historical	Static Factor
$RMSE^{\text{ind}}$	1.5760	2.5679	2.6629
$RMSE^{\text{avg}}$	0.2994	0.5805	0.6017
RC	0.4847	0.4702	0.4592

Table 5: Short term MES forecasting.

data. Here, we report diagnostics on the TARCH-DCC residuals series $(\hat{\epsilon}_{mt}, \hat{\xi}_{it})$ and focus on assessing the degree of dependence in the lower tails. To do this, we analyse the Hit_t variable, a binary indicator which is one when both residuals are simultaneously below their respective 10% quantile. Under the null of correct specification and absence of lower tail dependence, the sequence of hits behaves as an iid Bernoulli sequence with $p = 1\%$ and we can construct tests in the same spirit of Christoffersen (1998), Engle and Manganelli (2004) and Patton (2002) to asses if this hypothesis is valid. For each firm in the panel we perform a binomial test under the null that the hit sequence is generated by a Bernoulli with $p = 1\%$. Table 4 reports the average of the Hit_t variable together with the proportion of rejected tests at the 1% level by industry group. The hit average is always higher than 1% but the deviation appears to be mild and the binomial test is non significant in the majority of cases. Results indicate that the performance of the TARCH-DCC approach is satisfactory but there is indeed some evidence of neglected lower tail dependence. As far as MES forecasting this is not a concern as our prediction approach captures this source of residual dependence. It is important to stress however that residual dependence is marginal and that for time varying volatility and correlation account for most of the dependence between firm and market returns.

4.3   Short Term MES Prediction and Evaluation

In this section we carry out a short term MES forecasting exercise to assess the predictive ability of our specification against two simple benchmarks. The first one is the MES estimator
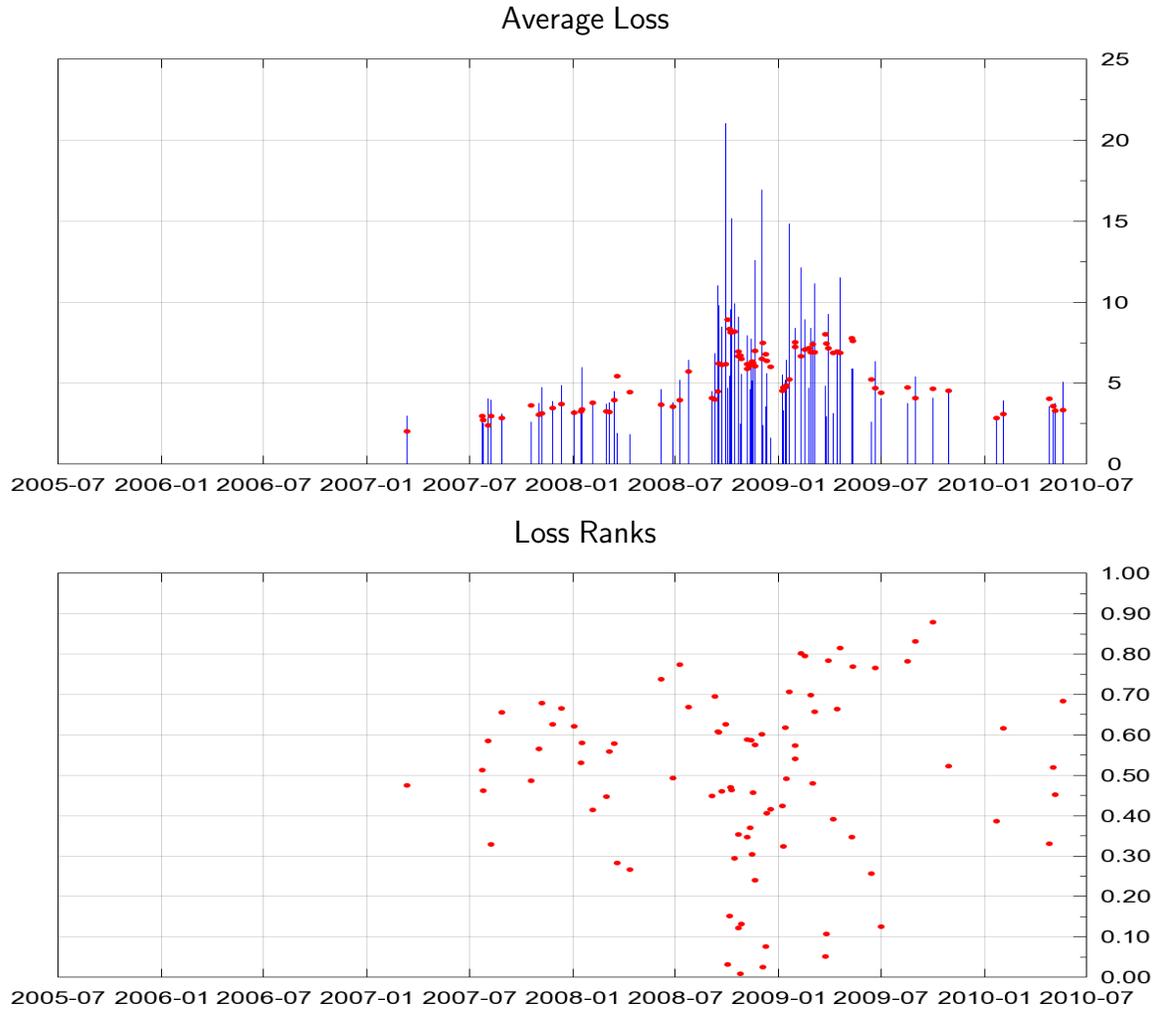
Figure 6: Short term MES forecasting.

proposed in Acharya *et al.* (2010), which we label as "Historical", defined as

$$\mathsf{MES}_{i\,t}^{1\,\mathsf{his}}(C) \equiv \frac{\sum_{\tau=t-W}^{t-1} r_{i\tau} I(r_{m\tau} < C)}{\sum_{\tau=t-W}^{t-1} I(r_{m\tau} < C)}, \tag{8}$$

that is the average of firm returns on event days over a given window of most recent observations (we use $W = 4$ years). It is inspired by the risk management practice where rolling averages are often used to obtain estimates of ES or VaR. The second MES estimator is the

one implied by assuming that firm returns are generated by a simple static factor model, i.e.

$$r_{it} \;\; = \;\; \beta_i \, r_{mt} + \epsilon_{it},$$

where the variance of the idiosyncratic innovation ϵ_{it} is assumed to be constant. The "Static Factor" MES benchmark is then defined as

$$\mathsf{MES}_{it}^{1\,\mathsf{sf}}(C) \equiv \hat{\beta}_i \, \widehat{ES}_m(C), \tag{9}$$

where $\hat{\beta}_i$ is the least square estimator of the factor loading and $\widehat{ES}_m(C)$ is the historical estimator of the ES of the market. Both statistics are computed with a rolling window estimation scheme using the last 4 years of data available.

We introduce a number of different metrics to evaluate forecasts. All metrics evaluate predictive ability by comparing on event days (i.e, when $r_{mt} < C$) the predicted MES with the realised losses, that is

$$\mathsf{MES}_{it}(C) \quad \text{and} \quad L_{it} = -r_{it}.$$

The first evaluation metric we report is the Relative Mean Square Error of all individual MES predictions

$$RMSE^{\mathsf{ind}} = \frac{1}{I N_c} \sum_{it} \left(\frac{\bar{L}_{it} - \mathsf{MES}_{it}(C)}{\mathsf{MES}_{it}(C)} \right)^2,$$

where N_c is the total number of event days in the sample. We opt for a Relative MSE rather then absolute in that the series of losses exhibits strong heteroskedasticity due to time varying volatility. Standardizing by the MES predictions allows us to make a more fair comparison. It avoids realized losses on volatile periods to essentially determine the value of this performance measure.

We also report the discrepancy between the average loss of all the firms in the system an

the predicted one. Let

$$\overline{\text{MES}}_t = \frac{1}{I} \sum_{i=1}^{I} \text{MES}_{it}(C) \quad \text{and} \quad \bar{L}_t = \frac{1}{I} \sum_{i=1}^{I} L_{it}$$

denote the predicted and actual average loss, We compute the Relative Mean Square Error (RMSE) of the average loss as

$$RMSE^{\text{avg}} = \frac{1}{N_c} \sum_t \left(\frac{\bar{L}_t - \overline{\text{MES}}_t}{\overline{\text{MES}}_t} \right)^2 .$$

The average rank correlation between MES predictions and the actual losses is the last evaluation metric we propose. Apart from the ability to produce accurate forecasts of the system average losses, MES predictions could also be of great interest if able to successfully rank firms according to the severity of the individual losses. The Rank Correlation (RC) is computed as the Pearson correlation coefficient between ranks, that is

$$RC_t = \frac{\sum (R_i^L - \bar{R}^L)(R_i^{\text{MES}} - \bar{R}^{\text{MES}})}{\sqrt{\sum (R_i^L - \bar{R}^L) \sum (R_i^{\text{MES}} - \bar{R}^{\text{MES}})}}$$

where R_{it}^L and R_{it}^{MES} are the loss and MES ranks for firm i. We denote the average rank correlation by RC.

The forecasting exercises are implemented as follows. Starting from July 2005 we compute 1 period ahead MES predictions conditionally on a 2% market drop. The parameter estimates needed to produce forecasts from the GARCH/DCC/Nonparametric approach are updated once a week on the last weekday.

Figure 6 displays summary plots of the short term MES forecasts. There are 76 event days in between July 2005 and July 2010, most of which are concentrated between the end of 2008 and the beginning of 2009. The average losses are typically between 2% and 5% but at the peak of the crisis they are as high as 20%. The time series profile of the average losses is, as expected, the same as volatility. MES tracks average losses well most of the time with the

exception of the crisis climax, where forecasts tend to underestimate the actual losses. Rank correlation does not have a clear time series pattern and is on average moderately high, being around 50%.

Table 5 reports summary statistics of the short term MES forecasts against the two benchmarks. All metrics show that the dynamic approach we propose performs systematically better. A Diebold Mariano predictive ability test finds that our approach produces significantly better individual and average loss forecasts at the 5% level.

4.4   Long Run MES Prediction and Systemic Risk

The forecasting exercise of the previous section shows the predictive ability of our model in forecasting short term downside exposure to market shocks. While these forecasts give important signals, they are of limited use from a systemic risk measurement perspective. Firstly, the horizon of interest of a regulator is generally speaking a long one and by construction short term measures give greater importance to recent shocks rather than long run dynamics. Secondly, following the Acharya *et al.* (2010) framework, MES alone does not convey which firms are big systemic risk contributors. To overcome these limitations, in this section we forecast long run MES to compute the SRISK index proposed in Section 2.

We design our long run forecasting methodology as follows. At the end of each month starting from July 2005, we estimate our specification for each firm in the panel that has been trading for at least 2 years. Using the simulation based procedure described in Section 3, we compute the long run MES of each firm and ES of the market conditionally on a market drop of 40% over the following six months, as well as the time varying probability of the systemic event PoS. Using debt and equity data we then compute the expected capital shortfall and the SRISK measures. Finally, we use SRISK to construct rankings of the top ten systemically risky institutions. Our methodology allows one to compute daily rankings and we chose a monthly update frequency only to reduce the computational burden. In the computation of
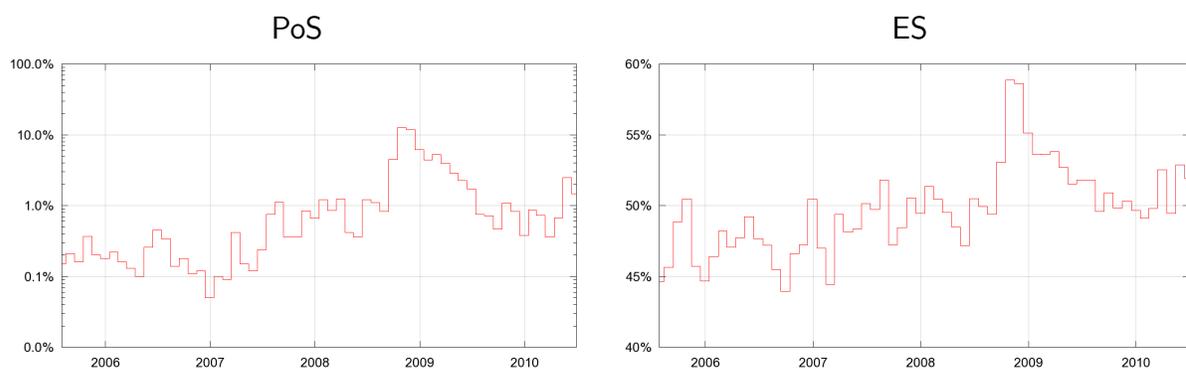
Figure 7: Long run probability of a systemic event (PoS) and Expected Shortfall (ES) predictions.

the expected capital shortfall, debt is proxied with its book value, equity is proxied with the market cap and the prudential ratio k is set to 8%.

The long run predictions of PoS and ES give insights on the model implied views on the long run risks of the economy. Figure 7 shows the two series between 2005 and 2010. In 2005 the level of PoS is moderately low, being roughly 0.1%. However, starting from 2007 the probability escalates rather steeply and by the end of the year it is increased by 10 times. PoS reaches its peak at 10% after the Lehman bankruptcy and it then decreases and stabilizes around 1% starting from mid 2009. The ES ranges between 45% and 50% until the Lehman bankruptcy where it spikes up to almost 60%, implying at the end of the fall of 2008 a crisis is not just more likely but also expected to be more severe. ES then regresses to a lower level and by mid 2010 is approximately 50%.

Long run MES and CS show the exposure and capital shortage of an individual firm to be expected in case of a systemic event. Rather than looking at industry averages, it is interesting to look at individual firm results. In Figure 8 we report the results of a representative firm from each group: Citigroup, AIG, Lehman and Fannie. The Lehman and Fannie series are interrupted in September 2009. Over all, all long run MES predictions display a similar time trend. MES starts increasing from January 2007, it peaks at the end of 2008/beginning of 2009 and then declines and stabilizes at lower levels that, however, are still higher than those
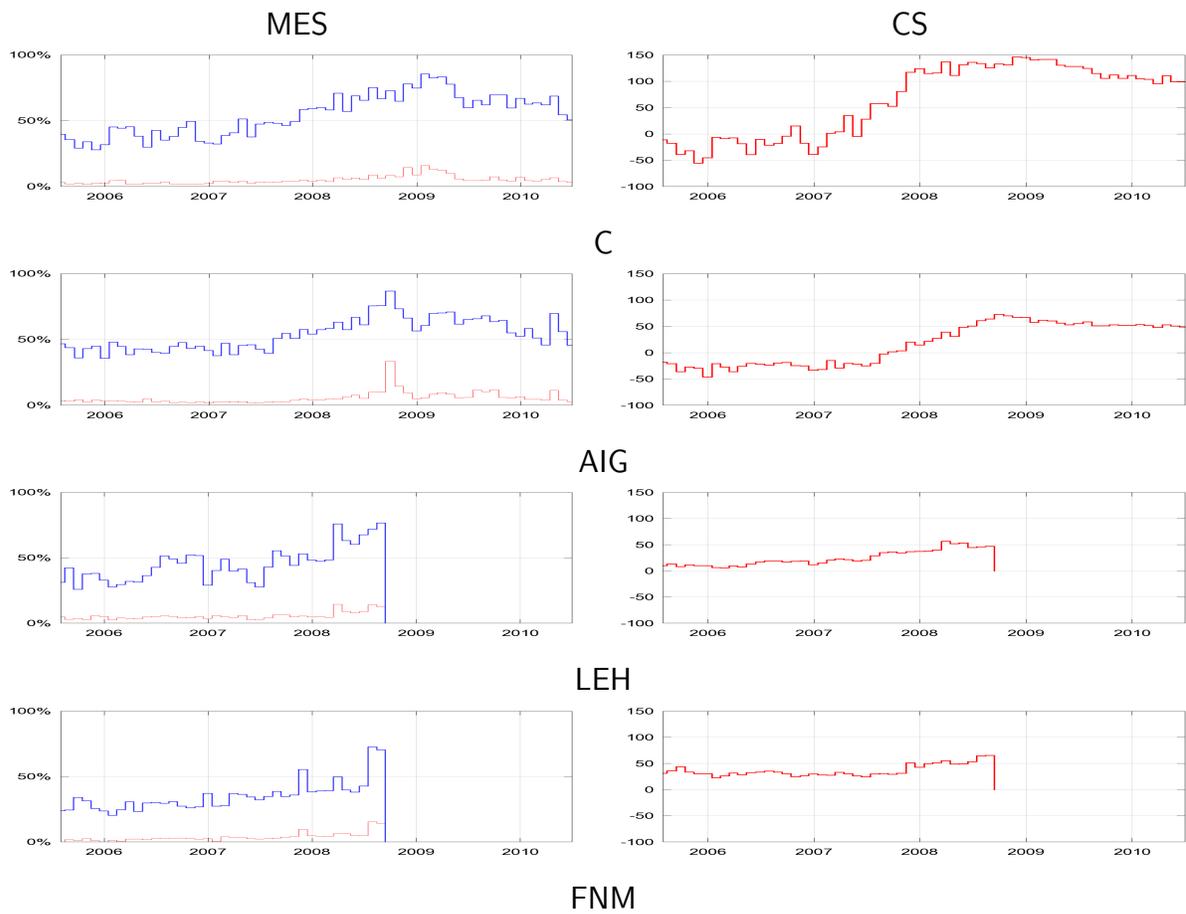
Figure 8: Long run MES predictions and Expected Capital Shortfall CS.

observed in the before the beginning of the crisis. The peaks of the series differs from firm to firm: some firms peak in September 2009 while others in February 2010, before the beginning of the market recovery. The capital shortfall estimates give some interesting insights on the shortage dynamics in this period. In the pre-crisis period Lehman and Fannie have a large capital shortfall while Citigroup and AIG are appropriately capitalized (they have a negative shortage, that is they have enough capital in case of a crisis). Starting from January 2007 capital shortfall begins to quickly deteriorate. Before Lehman files for bankruptcy and Fannie is place under conservatorship in September 2009, their capital shortage have, respectively, quintupled and almost tripled with respect to the initial levels. Citigroup and AIG also have a sharp increase in capital shortage which is particularly severe for Citigroup, which by January
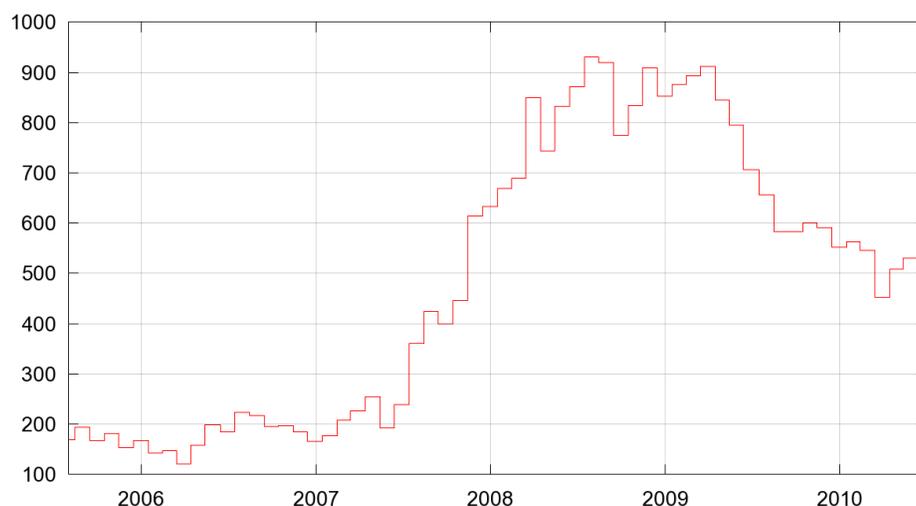
Figure 9: Aggregate SRISK of the top U.S. financial institutions.

2008 exceeds 100 USD billions.

The aggregate SRISK summarises the capital shortage dynamics in the whole system. It is defined as the sum of all the positive capital shortfalls of all firms and its plot is displayed in Figure 9. In the pre crisis period the total shortfall is estimated to be around 200 USD billions. This figure rapidly escalates starting from 2007 and starting from January 2008 is already tripled. The series reaches of 900 USD billions as a result of the Lehman bankruptcy and starts decreasing only with the start of the market rally in March 2009. As of July 2010 the system still looks not fully recovered.

Finally, in Table 6 we report the rankings of the systemically risky U.S financial institutions. On eight pre selected dates we report the top 10 most risky institutions according to SRISK% measure. At the beginning of 2007, when the total capital shortfall of the system is modest, the list contains mostly highly levered Broker-Dealers firms together with Freddie and Fannie. As the crisis unwinds, large commercial banks start rising up in the top ten, like Citigroup, Bank of America and JP Morgan. It is interesting to stress that as of March 2007, approximately 1 year and a half before the Lehman bankruptcy, 8 firms out of the SRISK top

Rank	2007-03-30		2007-06-29		2007-12-31		2008-02-29	
1	MS	24.3%	MS	19.0%	C	19.6%	C	16.8%
2	FNM	14.5%	FRE	14.4%	MS	9.9%	MS	8.9%
3	FRE	14.3%	MER	12.7%	MER	9.2%	MER	8.6%
4	MER	10.6%	C	11.8%	FRE	8.6%	FRE	7.6%
5	LEH	10.1%	FNM	10.1%	FNM	6.7%	FNM	7.5%
6	BSC	8.7%	BSC	8.8%	GS	6.5%	JPM	7.2%
7	GS	7.5%	LEH	8.3%	JPM	6.1%	GS	6.9%
8	MET	3.2%	GS	5.7%	LEH	5.9%	LEH	5.8%
9	HIG	2.7%	MET	3.5%	BSC	3.9%	AIG	3.9%
10	C	1.7%	HIG	3.1%	WM	3.1%	WB	3.9%

Rank	2008-06-30		2008-08-29		2009-01-30		2010-06-30	
1	C	15.6%	C	13.6%	C	16.1%	C	17.5%
2	BAC	8.5%	JPM	9.3%	JPM	15.7%	BAC	17.3%
3	MS	7.7%	BAC	8.8%	BAC	14.1%	JPM	15.1%
4	JPM	7.4%	FRE	7.4%	WFC	8.4%	AIG	8.7%
5	MER	7.4%	FNM	7.1%	AIG	6.6%	MS	8.2%
6	FRE	6.5%	MER	7.1%	GS	5.7%	MET	4.9%
7	FNM	6.2%	MS	7.0%	MS	4.9%	GS	4.1%
8	AIG	5.8%	AIG	6.9%	MET	3.3%	PRU	3.8%
9	LEH	5.1%	GS	5.5%	PRU	3.2%	HIG	3.4%
10	GS	5.0%	WB	5.1%	HIG	2.4%	WFC	3.3%

Table 6: Systemic Risk rankings

10 are all institutions that in different ways have been severely hit by the crisis and that have produce negative externalities to the economy.

5   Conclusion

The 2007/2009 financial crisis calls for a better understanding of systemic risk. In this paper we propose a systemic risk index called SRISK which measures the expected capital shortfall of a financial institution in a crisis. Following Acharya *et al.* (2010), capital shortfall in a crisis measures systemic risk in that such shortage is going to generate a negative externalities to the whole economy. The argument is that an institution in distress when the economy is already in distress, cannot be taken over by the rest of the system and the government faces the question of whether to rescue the firm with taxpayer money as other avenues are no longer available. SRISK is a function of the level of leverage of the firm and MES, the tail expectation of the firm equity returns conditionally on a substantial loss in the market. While the leverage of a firm can be measured using balance sheet data, MES requires appropriate time series methodology. We develop an econometric model for the bivariate firm and market return which decomposes the dynamics of the pairs in terms of time varying volatility, correlation and possibly nonlinearly dependent innovation shocks. The model allows one to easily construct short and long run MES predictions. We analyse the systemic risk of top U.S. financial firms between 2005 and 2010. MES analysis provides useful tools for monitoring systemic risk and, in retrospect, it captures several of the early signs of the financial crisis.

"Credit Risk, Systemic Risk, and Large Portfolios" Venice, 30 September - 1 October, 2010; "47th Annual Conference on Bank Structure and Competition", Chicago, May 4-6, 2011; "Nonlinear and Financial Econometrics Conference a Tribute to A. Ronald Gallant" Toulouse, May 19-21, 2011 and "Interdisciplinary workshop on Econometric and Statistical Modelling of Multivariate Time Series", Louvain-la-Neuve, May 25-27, 2011. All mistakes are ours.

References

Acharya, V., , Pedersen, L., Philippe, T., and Richardson, M. (2010). Measuring systemic risk. Technical report, Department of Finance, NYU.

Adrian, T. and Brunnermeier, M. K. (2009). CoVaR. Technical report, FRB of New York. Staff Report No. 348.

Aielli, G. (2006). Dynamic conditional correlations: On properties and estimation. Technical report, Department of Statistics, University of Florence.

Aielli, G. (2009). Dynamic conditional correlations: On properties and estimation. Technical report, Department of Statistics, University of Florence.

Allen, L., Bali, T. G., and Tang, Y. (2010). Does systemic risk in the financial sector predict futur economic downturns? Technical report.

Billio, M., Getmansky, M., Lo, A. W., and Pellizzon, L. (2010). Econometric measures of systemic risk in the finance and insurance sectors. Technical report.

Bollerslev, T. (2008). Glossary to arch (garch). Technical report, Glossary to ARCH (GARCH). Research Paper 2008-49.

Brownlees, C. T. (2011). On the relation between firm characteristics and volatility dynamics with an application to the 2007-2009 financial crisis. Technical report.

Caporin, M. and De Magistris, P. S. (2011). On the Evaluation of Marginal Expected Shortfall. *Applied Economics Letters*, (forthcoming).

Christoffersen, P. F. (1998). Evaluation of interval forecasts. *International Economic Review*, **39**, 841–862.

Ding, Z. and Engle, R. (2001). Large scale conditional covariance matrix modeling, estimation and testing. Technical report, NYU Stern School of Business. Working Paper FIN-01-029.

Engle, R. (2002a). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics*, **20**(3), 339–350.

Engle, R. (2009). *Anticipating correlations: a new paradigm for risk management*. Princeton University Press.

Engle, R. and Manganelli, S. (2004). Asset allocation by variance sensitivity analysis. *Journal of Business and Economic Statistics*, **22**, 367–381.

Engle, R. and Sheppard, K. (2001). Theoretical and empirical properties of dynamic conditional correlation multivariate garch. Technical report, NBER. Working Paper No. W8554.

Engle, R., Shephard, N., and Sheppard, K. (2009). Fitting vast dimensional time-varying covariance models. Technical report, NYU. Working Paper No. FIN-08-009.

Engle, R. F. (2002b). New frontiers for ARCH models. *Journal of Applied Econometrics*, **17**, 425–446.

Glosten, L. R., Jagananthan, R., and Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance*, **48**(5), 1779–1801.

Hautsch, N., Schaumburg, J., and Schienle, M. (2010). Quantifying time–varying marginal systemic risk contributions. Technical report.

Manganelli, S., Kim, T.-H., and White, H. (2010). Var for var: Measuring systemic risk using multivariate regression quantiles. Technical report.

Mezrich, J. and Engle, R. F. (1996). GARCH for groups. *Risk*, **9**, 36–40.

Rabemananjara, R. and Zakoïan, J. M. (1993). Threshold ARCH models and asymmetries in volatility. *Journal of Applied Econometrics*, **8**(1), 31–49.

Scaillet, O. (2004). Nonparametric estimation and sensitivity analysis of expected shortfall. *Mathematical Finance*, **14**, 115–129.

Scaillet, O. (2005). Nonparametric estimation of conditional expected shortfall. *Insurance and Risk Management Journal*, **74**, 639–660.

Scaillet, O. and Fermanian, J. D. (2005). Sensitivity analysis of var and expected shortfall for portfolios under netting agreements. *Journal of Banking and Finance*, **29**, 927–958.

Schwaab, B., Koopman, S.-J., and Lucas, A. (2011). Systemic risk diagnostics, coincident indicators and early warning signals. Technical report.

Tasche, D. (2000). Risk contributions and performance measurement. Technical report.

Taylor, J. W. (2008). Using exponentially weighted quantile regression to estimate value at risk and expected shortfall. *Journal of Financial Econometric*, **6**, 382–406.